Bayesian Estimation of Between-Case Standardized Mean Differences: A Simulation Study

James E. Pustejovsky (pustejovsky@wisc.edu), Man Chen, David A. Klingbeil, & Ethan R. Van Norman

School of Education
UNIVERSITY OF WISCONSIN–MADISON

Overview

- The between-case standardized mean difference (BC-SMD) for single-case designs is a study-level summary effect size measure that describes an *average* intervention effect on a scale that is comparable to a SMD from a between-group design conducted under the same operational conditions.
- Existing tools for calculating BC-SMDs (*scdhlm* R package and Shiny app) use REML methods to estimate a hierarchical model for the data, the parameters of which are then combined to form a BC-SMD estimate. However, REML estimation often fails to converge for studies with a limited number of cases or for more complex hierarchical models (i.e., models with multiple random effects terms).
- The purpose of this study is to use Monte Carlo simulation to investigate the potential of Bayesian methods to improve the estimation of BC-SMD effect sizes, comparing Bayesian methods to the currently available method of REML estimation. Our simulation considered **MB2** model in Pustejovsky et al. (2014) which has the form:

$$Y_{ti} = \beta_{0i} + \beta_{1i}B_{ti} + e_{ti}, \quad \text{Var}(e_{ti}) = \sigma^2, \; cor(e_{si}, e_{ti}) = \phi^{|t-s|}$$
$$\beta_{0i} = \gamma_{00} + \eta_{0i} \qquad (\eta_{0i}, \eta_{1i}) \sim MVN\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{bmatrix}\tau_0^2 & \psi\tau_0\tau_1 \\ \psi\tau_0\tau_1 & \tau_1^2\end{bmatrix}\right)$$
$$\beta_{1i} = \gamma_{10} + \eta_{1i}$$

- BC-SMD: $\delta_{AB} = \gamma_{10} / \sqrt{\tau_0^2 + \sigma^2}$

Bayesian priors

- Fixed effect parameters and variance component (SDs) are defined on the same scale as the outcome variable. Thus, prior specifications need to consider the outcome scale.
- Prior to fitting the model, we standardized the outcome variable using $\tilde{Y}_{ti} = \frac{1}{s}\left(Y_{ti} - \frac{1}{m}\sum_{i=1}^{m} b_{0i}\right)$.
- We examined Bayesian estimators with non-informative (default) priors and with stronger, informative priors on the variance components.

Non-informative	*Informative*
$\gamma_0 \sim Uniform(-\infty, \infty)$	$\gamma_0 \sim Uniform(-\infty, \infty)$
$\gamma_1 \sim Uniform(-\infty, \infty)$	$\gamma_1 \sim Uniform(-\infty, \infty)$
$\tau_0 \sim t^+(3)$	$\tau_0 \sim \Gamma(6,11)$
$\tau_1 \sim t^+(3)$	$\tau_1 \sim \Gamma(6,11)$
$\sigma \sim t^+(3)$	$\sigma \sim \Gamma(4,4)$
$\psi \sim Uniform(-1, 1)$	$\psi \sim Uniform(-1, 1)$
$\phi \sim Uniform(-1, 1)$	$\phi \sim N(0.2, 0.1)$

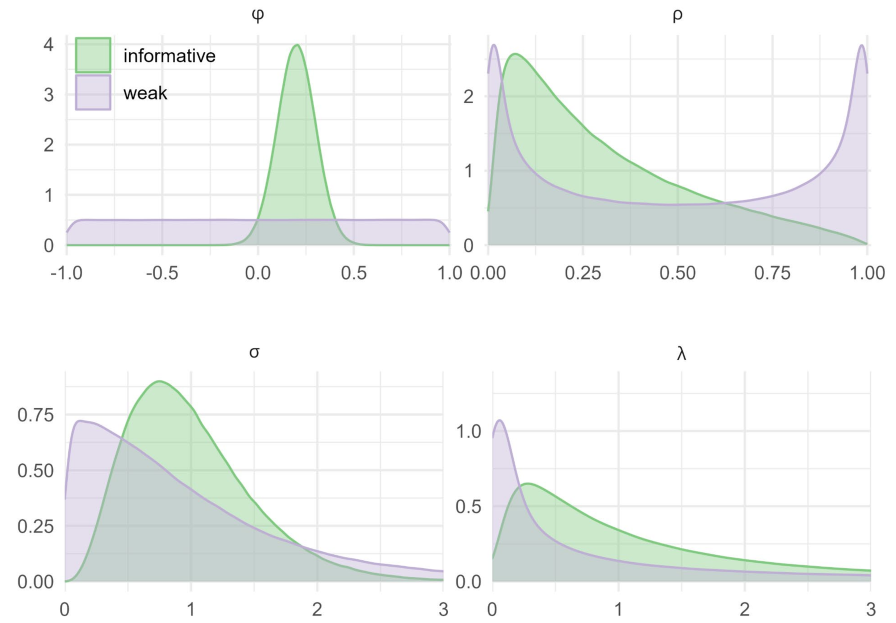


Figure 1. Default (weak) and informative priors for auto-correlation (ϕ), intra-class correlation (ρ), within-series standard deviation (σ), and relative variance of the treatment effects (λ).

Simulation Methods

Data Generation

- Generated baseline phase lengths based on m and N.
- Simulated raw observations for each case based on the MB2 model.
- Calculated the REML estimator of the BC-SMD using the *scdhlm* R package. Calculated the weak and informative Bayesian estimators using the *brms* package.

Simulation Design

Parameter	Values
Number of cases (m)	3, 4, 6, 8
Number of measurement occasions (N)	20, 30, 40
True treatment effect (δ_{AB})	0, 0.5, 1, 2
Intra-class correlation (ρ)	0, 0.2, 0.4, 0.6
Relative variance of the true effect (λ)	0, 0.2, 0.4
Auto-correlation (ϕ)	0, 0.2, 0.4, 0.6

Figure 2. Relative bias of BC-SMD estimators. The informative Bayesian estimator has similar (or even slightly better) performance in terms of bias compared to REML estimator when the priors are in the vicinity of the generating parameter values.

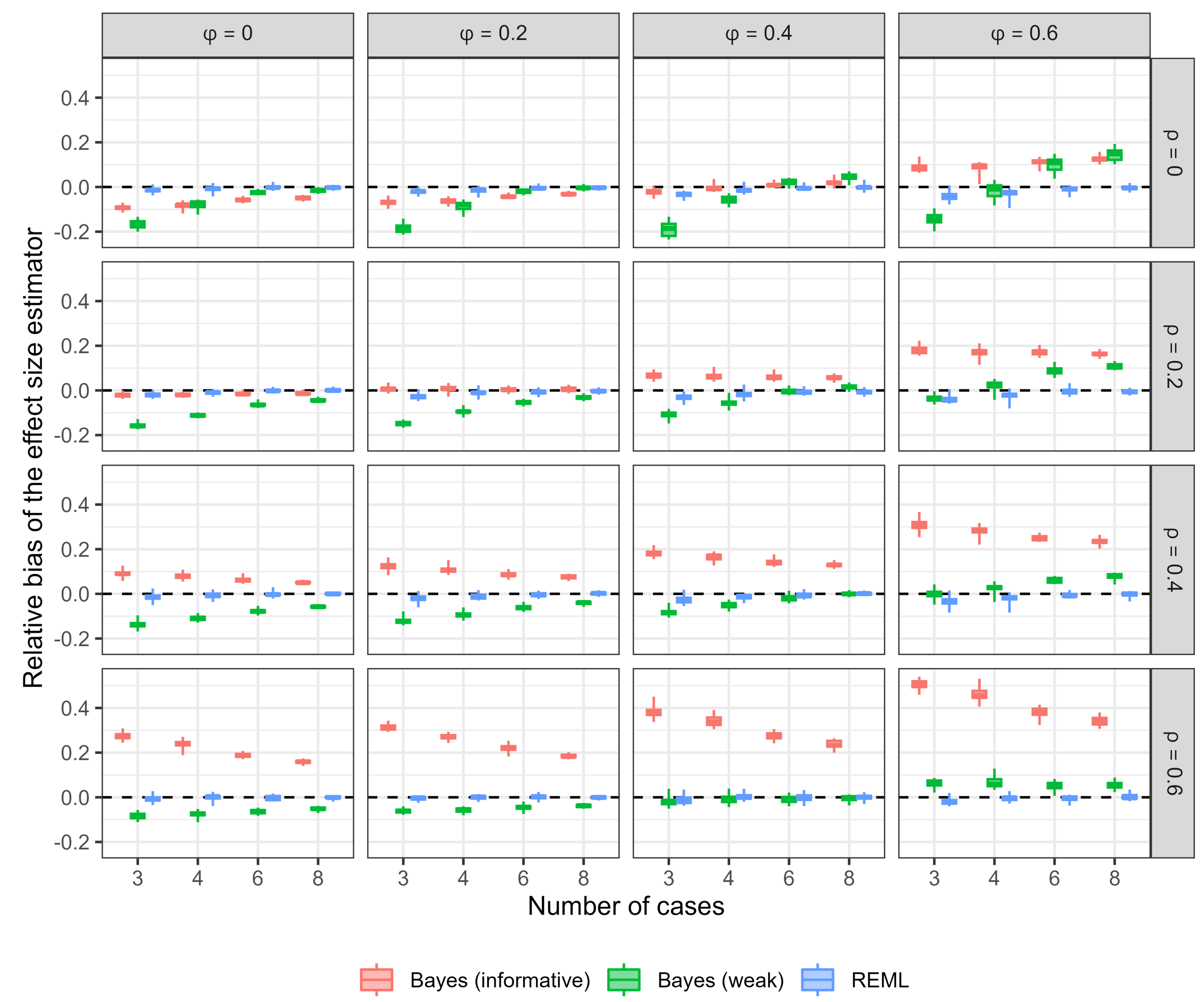


Figure 3. Relative RMSE of BC-SMD estimators. When the parameter values of ϕ and ρ are within the range supported by the informative priors, the informative Bayesian estimator has accuracy that rivals REML (and sometimes even slightly more accurate) and better accuracy than the weak Bayesian estimator.

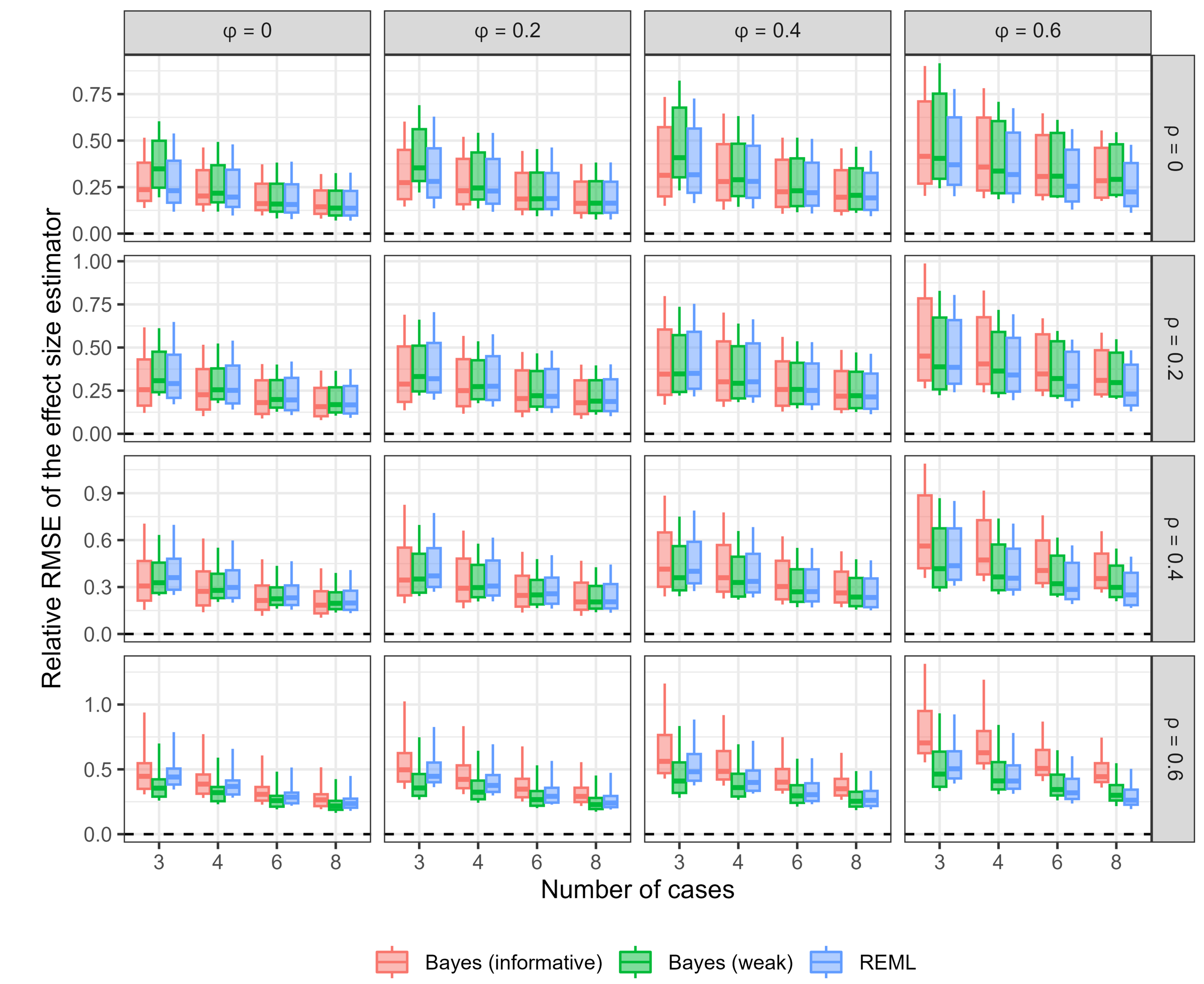


Figure 4. Confidence interval coverage of BC-SMD estimators. Both REML and weak Bayesian estimators tend to over-cover, although the REML intervals generally have closer to nominal coverage rates. The informative Bayesian tends to over-cover for small ϕ and ρ but tends to have coverage rates below (and sometimes far below) nominal when the parameters values are not supported by the informative priors.

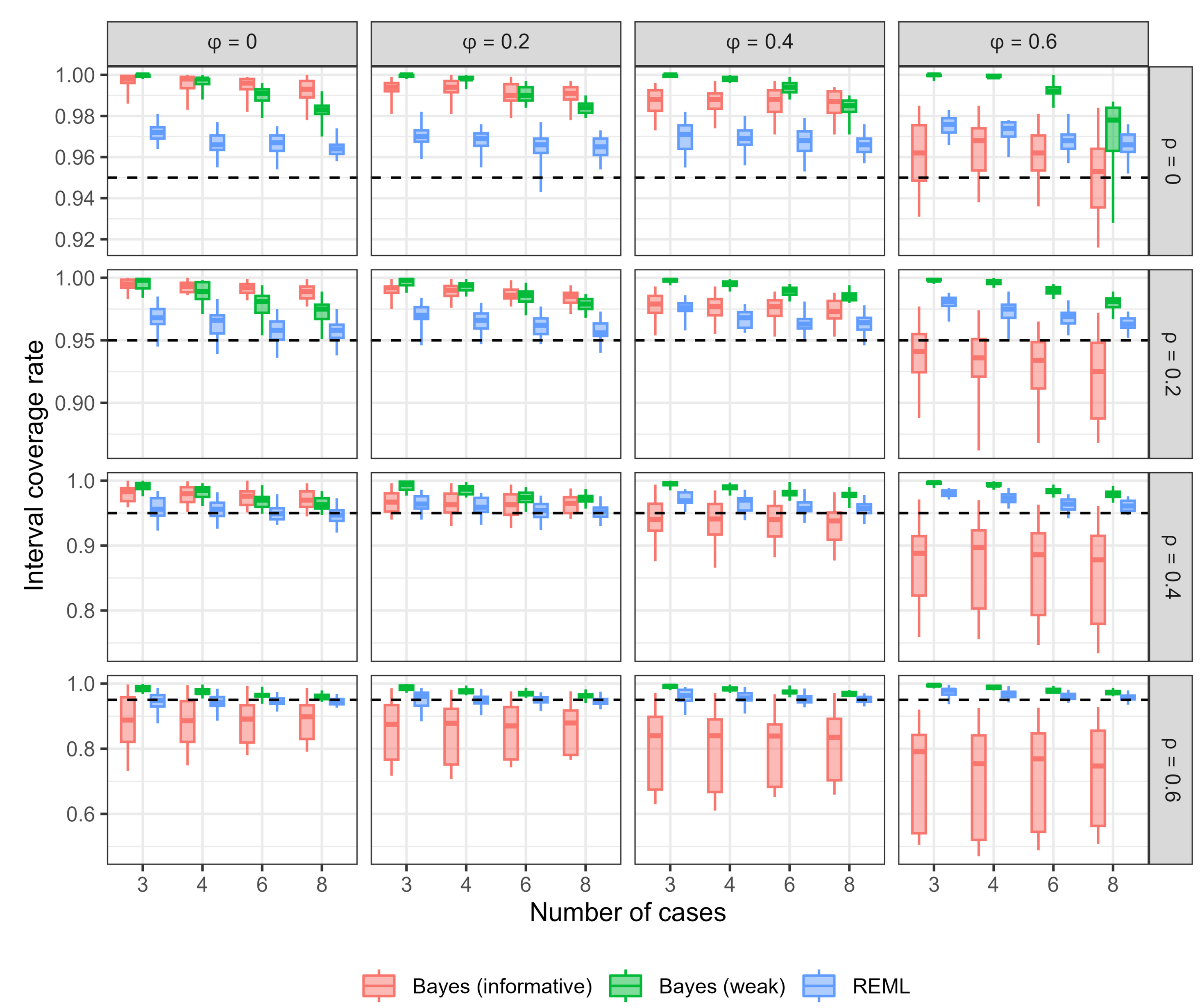
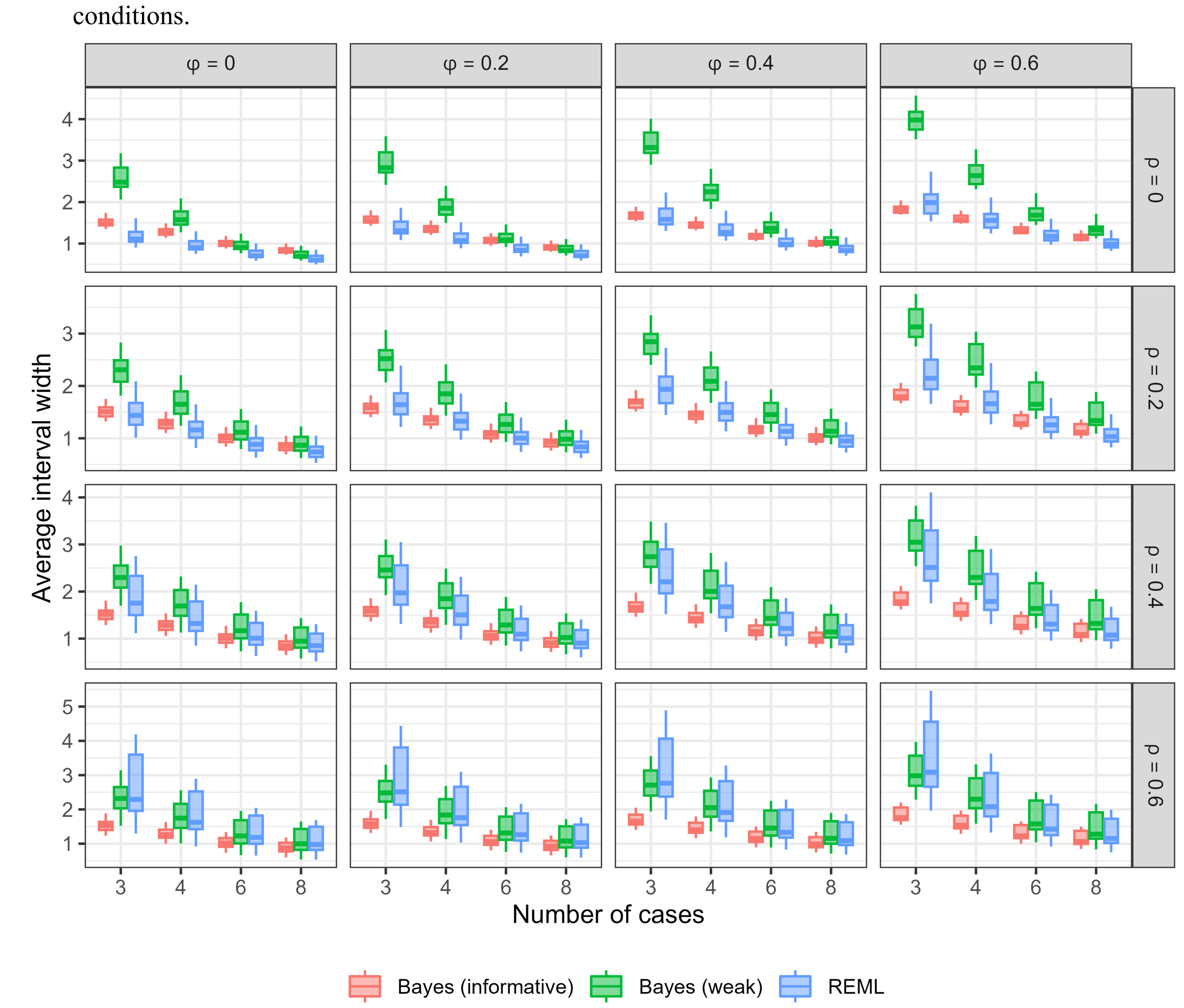


Figure 5. Confidence interval width of BC-SMD estimators. The informative Bayesian intervals have similar or narrower width than the intervals from other estimation methods, particularly for high ρ. However, the informative Bayesian intervals have coverage far below nominal levels under these conditions.



The research reported in this article was supported by Grant R305D160002 from the Institute of Education Sciences, U.S. Department of Education. The opinions expressed are those of the authors and do not represent the views of the Institute or the U.S. Department of Education.