Empirical Benchmarks for Between-Case Standardized Mean Differences from Single-Case Multiple Multiple Baseline Designs Examining Academic Interventions

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Outline

- Single-case multiple baseline designs
- Between-case standardized mean differences
- Systematic review of academic multiple baseline designs
- Analytic strategy
- Empirical benchmarks
Single-case multiple baseline designs

• Used for investigating effects of interventions / practices for individual participants across a variety of settings

• Essential features of multiple baseline designs
  • One or small number of participants
  • Repeated measurement of outcomes on each individual participant
  • Researcher controls introduction of intervention for each participant
  • Intervention initiation is staggered in time
Rodriguez & Anderson (2014). Integrating a social behavior intervention during small group academic instruction using a total group criterion intervention.
Design schematic

A multiple baseline design across four participants

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The design schematic shows a multiple baseline across four participants (T1 to T18), with each participant undergoing two phases: a baseline (X) and an intervention (T). The design switches between participants in a staggered manner, allowing for the observation of effects within and across participants.
Outline

• Single-case multiple baseline designs
• **Between-case standardized mean differences**
• Systematic review of multiple baseline designs
• Analytic strategy
• Empirical benchmarks
Between-case standardized mean difference (BC-SMD)

• Shadish, Rindskopf, & Hedges (2008) asked:

  \textit{Can we estimate an effect size based on the data from a single-case design that is in the same metric as the standardized mean difference effect size from a between-groups design?}

• Why do this? (Shadish, Hedges, Horner, & Odom, 2015)
  • \textbf{Translation} of single-case research for researchers who work primarily with between-groups designs.
  • \textbf{Comparison} of results from single-case studies and between-groups studies, for purposes of understanding the utility and limitations of each type of design.
  • \textbf{Synthesis} involving both single-case and between-groups designs.
SMD in between-group experiments

• What is the SMD from a between-groups experiment?

\[
\delta_{BC} = \frac{\text{Average outcome if everybody gets treatment} - \text{Average outcome if nobody gets treatment}}{\text{Outcome standard dev.}}
\]

\[
\delta_{BC} = \frac{\text{Average outcome if everybody gets treatment} - \text{Average outcome if nobody gets treatment}}{\sqrt{\text{Within participant variance} + \text{Between participant variance}}}
\]

• These quantities can be estimated from multiple baseline design data using a hierarchical linear model.
  • We’ll need to have a sample of multiple participants (bare minimum of 3, more for more complex models).
  • We’ll need to be specific about timing of intervention and follow-up.
Estimating BC-SMDs: The broad strategy

Pustejovsky, Hedges, and Shadish (2014):

1. Develop a hierarchical linear model that describes
   a) the form of time trends and intervention effects
   b) how the trends and intervention effects vary across cases.

2. Imagine a hypothetical between-groups experiment with the
   same population of participants, same treatment, same
   outcomes.
   • When is treatment initiated?
   • When are outcomes assessed?

3. Use the hierarchal model to estimate the between-case SMD for
   the hypothetical experiment.
Design translation

A multiple baseline design

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A hypothetical between-group design (with pre-test)

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Motivation

• BC-SMD has been used in many primary single-case design studies, as well as many systematic reviews of single-case research.

• What Works Clearinghouse recently adopted BC-SMDs for summarizing findings from single-case designs.

• But no reference benchmarks available.
  • Theoretically comparable to between-group effect sizes.
  • But multiple baselines are used in different contexts that group designs, so existing group design benchmarks are probably not be appropriate.
Outline

- Single-case multiple baseline designs
- Between-case standardized mean differences
- Systematic review of multiple baseline designs
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- Empirical benchmarks
Inclusion criteria and search strategy

• **Design:** Across-participant multiple baseline design with 3+ participants.

• **Participants:** Students in pre-kindergarten through 12th grade (or Special Education up to age 21)

• **Intervention:** Any intervention targeting an academic skill

• **Comparison:** Baseline prior to intervention

• **Outcomes:** Specific, curriculum-based measures of math, reading, or writing

• **Databases:** Academic Search Complete, ERIC, PsycInfo

• **Search string:** “single-case” AND (“read*” OR “math*” OR “writ*” OR “spell*” OR “academic*” OR “learn*”)
Records identified through database searching (n = 24,238)

Records identified through other sources (n = 54)

Records after duplicates removed (n = 9,867)

Titles/abstract excluded (n = 8,051)

Initial Exclusion (n = 879):
Not SCED (n = 241)
Not academic intervention (n = 575)
No child/student outcomes (n = 60)
Main text not in English (n = 3)

Full-text articles assessed for eligibility (n = 1,844)

Full-text articles assessed for eligibility: Reading (n = 500)
Excluded (n = 421)
• Ineligible design (n = 271)
• Outcome not words read correct per unit of time (n = 150)

Records included for Reading (n = 79)

Full-text articles assessed for eligibility: Math (n = 255)
Excluded (n = 209)
• Ineligible design (n = 154)
• Outcome not computation problems correct, items correct, or digits correct (n= 55)

Records included for Math (n = 46)

Full-text articles assessed for eligibility: Writing (n = 210)
Excluded (n = 144)
• Ineligible design (n = 90)
• Outcome not TWW, WSC, CWS, C-IWS, on writing tasks (n = 54)

Records included for Writing (n = 66)
Outline

• Single-case multiple baseline designs
• Between-case standardized mean differences
• Systematic review of multiple baseline designs
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• Empirical benchmarks
Estimating BC-SMD effect sizes

• Initial visual analysis of every included design to determine appropriate functional forms for modeling.

• Effect size estimates generated using scdhlm R package (Pustejovsky, Chen, & Hamilton, 2022) based on a model with
  • Linear time trends for baseline phases
  • Intervention-by-time interactions
  • Random intercepts (but no random slopes)
  • Auto-correlated errors (AR1)

• Hypothetical between-group design parameters (defaults)
  • Intervention time equal to actual intervention time for first participant
  • Follow-up time based on the length of the shortest intervention phase
Summarizing distribution of effect sizes

- Empirical distribution of effect size estimates

- Meta-analytic model
  - Multi-level random effects
  - Prediction intervals for center of distribution

- Distribution of Empirical Bayes estimates
  - Non-parametric bootstrap intervals
Outline

• Single-case multiple baseline designs
• Between-case standardized mean differences
• Systematic review of multiple baseline designs
• Analytic strategy
• Empirical benchmarks
Empirical densities
## Meta-analysis and empirical Bayes

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Reference benchmarks

• Using middle 40% of distribution (30^{th}-70^{th} percentile)
• empirical Bayes estimates

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Observations and limitations

• Compared to group designs, distributions of BC-SMD effects from single-case multiple baseline designs cover *substantially larger values* and are *more dispersed*.

• Differences between designs could be due to differences in
  • Populations
  • Interventions
  • Dependent variables
  • Settings
  • Temporal horizons

• BC-SMDs from multiple baseline designs are sensitive to follow-up time.

• Critical to interpret findings within the context of the topic area and based on the logic of single-case designs.


