COMBINING ROBUST VARIANCE ESTIMATION WITH MODELS FOR DEPENDENT EFFECT SIZES
Many methods available for meta-analyzing dependent effect size estimates.
- ad hoc methods (Hammering the screws)
- model-based methods
- robust variance estimation (RVE)

Useful to combine RVE with model-based approaches.
- Addresses limitations of model-based approaches.
- Addresses limitations of default RVE implementation.
Model-based meta-analysis methods

Robust variance estimation

better together
DEPENDENT EFFECT SIZES ARE VERY COMMON

Multiple outcomes measured on common set of participants

<table>
<thead>
<tr>
<th>Treatment</th>
<th>O</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>O</td>
<td>P</td>
<td>Q</td>
</tr>
</tbody>
</table>

\[
d_{O1} \\
d_{P1} \\
d_{Q1}
\]

Outcome measured at multiple follow-up times

<table>
<thead>
<tr>
<th>Treatment</th>
<th>O_1</th>
<th>O_2</th>
<th>O_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>O_1</td>
<td>O_2</td>
<td>O_3</td>
</tr>
</tbody>
</table>

\[
d_{12} \\
d_{22} \\
d_{32}
\]

Multiple treatment conditions compared to a common control

<table>
<thead>
<tr>
<th>Treatment T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment U</td>
<td>O</td>
</tr>
<tr>
<td>Control</td>
<td>O</td>
</tr>
</tbody>
</table>

\[
d_{T3} \\
d_{U3}
\]

Multiple correlations from a common sample

\[
\begin{bmatrix}
  a & b & c & d \\
  a & r_{ab} & b & c & d \\
  b & r_{ab} & r_{bc} & b & c & d \\
  c & r_{ab} & r_{bc} & r_{cd} & c & d \\
  d & r_{ab} & r_{bc} & r_{cd} & r_{dd} & d
\end{bmatrix}
\]
33 experimental studies, 166 effect size estimates (standardized mean differences)

- Multiple outcomes (1-13 outcomes per study, median = 2)
- Multiple follow-up times (immediate post-test and/or later follow-up)
- Multiple treatment conditions (1-4 treatment conditions per study)
- Multiple control conditions (active and/or passive control)
- 1-52 effect size estimates per study (median = 2)
Covariances between ES estimates are often not available

- Multiple treatments compared to common control
  - known formulas (Gleser & Olkin, 2009), easy enough to calculate

- Multiple outcomes/multiple follow-ups
  - known formulas (Gleser & Olkin, 2009)
  - require knowing correlations among outcomes/repeated measures, which are not often reported

- Multiple correlations from common sample
  - known, icky formulas (Olkin & Siotani, 1976)
  - need to know correlations between ALL variables involved
BECKER (2000) DESCRIBED FOUR BROAD STRATEGIES FOR HANDLING DEPENDENCE:

- **Ignore**
  - Aggregated effects (Borenstein et al., 2009)

- **Combine**
  - Shifting unit-of-analysis (Cooper, 1998)

- **Sub-classify**
  - Sub-grouping

- **Model**
  - Multivariate meta-analysis (Raudenbush, Becker, & Kalaian, 1988; Kalaian & Raudenbush, 1996; Gleser & Olkin, 2009)
  - Robust variance estimation (Hedges, Tipton, & Johnson, 2010)
  - Meta-SEM (Cheung, 2014)
  - Multi-level meta-analysis (Van den Noortgate et al., 2013, 2015)
## COMPARISON

<table>
<thead>
<tr>
<th>Method</th>
<th>Requires making assumptions about ES covariances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated effects</td>
<td>✓</td>
</tr>
<tr>
<td>Sub-grouping</td>
<td>✓</td>
</tr>
<tr>
<td>Shifting unit-of-analysis</td>
<td>✓</td>
</tr>
<tr>
<td>Multivariate meta-analysis</td>
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<td>Multi-level meta-analysis</td>
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<tr>
<td>Meta-SEM</td>
<td>✓</td>
</tr>
<tr>
<td>Robust variance estimation</td>
<td>✓ (Working model)</td>
</tr>
</tbody>
</table>
Meta-analysis/meta-regression method using “sandwich” variance estimators, which are robust to mis-specified assumptions about variance-covariance structure.

Sandwich methods work with very general classes of models, including any of the other methods for handling dependent effects.

Proof: See Hedges et al. (2010, Appendix A).
## Comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>Requires making assumptions about ES covariances</th>
<th>Robustness to assumptions about ES covariances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated effects</td>
<td>✓</td>
<td>Robust*</td>
</tr>
<tr>
<td>Sub-grouping</td>
<td>✓</td>
<td>Robust*</td>
</tr>
<tr>
<td>Shifting unit-of-analysis</td>
<td>✓</td>
<td>Robust*</td>
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<td>✓ (Working model)</td>
<td>Robust</td>
</tr>
</tbody>
</table>

* When combined with robust (sandwich) variance estimation
DEFAULT RVE IMPLEMENTATION HAS LIMITATIONS
(Hedges, Tipton, & Johnson, 2010)

- Implementation in **robumeta** packages for R and Stata.
- Limited to two “working models”: correlated effects or hierarchical effects.
- Uses semi-efficient diagonal weights:

\[
    w_{ij} = \frac{1}{n_j (\bar{s}_j^2 + \hat{\tau}^2)}, \quad \text{where} \quad \bar{s}_j^2 = \frac{1}{n_j} \sum_{i=1}^{n_j} s_{ij}^2
\]

- Studies contributing more effects get less weight in meta-regressions that have within-study predictors.
- Similar to meta-regression after aggregating to the study level.
## Re-analysis of Self-control Training Studies

<table>
<thead>
<tr>
<th></th>
<th>(1) Aggregated effects</th>
<th>(2) Shifting unit-of-analysis</th>
<th>(3) Multivariate meta-analysis</th>
<th>(4) Multi-level meta-analysis</th>
<th>(5) Robust variance estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overall Average ES</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($k = 33, N = 166$)</td>
<td>0.281*** [0.059]</td>
<td></td>
<td>0.261*** [0.052]</td>
<td>0.263*** [0.054]</td>
<td>0.289*** [0.060]</td>
</tr>
<tr>
<td><strong>Between-study SD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.207</td>
<td></td>
<td>0.202</td>
<td>0.254</td>
<td>0.289</td>
</tr>
<tr>
<td><strong>Within-study SD</strong></td>
<td></td>
<td></td>
<td>0.143</td>
<td>0.027</td>
<td></td>
</tr>
</tbody>
</table>

### Moderator analysis by type of outcome

<table>
<thead>
<tr>
<th>Type</th>
<th>($k$, $N$)</th>
<th>(1) Aggregated effects</th>
<th>(2) Shifting unit-of-analysis</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Stamina</td>
<td>($16, 31$)</td>
<td>0.579*** [0.157]</td>
<td>0.413** [0.093]</td>
<td>0.359*** [0.077]</td>
<td>0.351*** [0.071]</td>
<td>0.579*** [0.123]</td>
</tr>
<tr>
<td>Strength</td>
<td>($28, 135$)</td>
<td>0.199** [0.071]</td>
<td>0.171** [0.064]</td>
<td>0.236*** [0.054]</td>
<td>0.238*** [0.055]</td>
<td>0.203** [0.065]</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>-0.380* [0.185]</td>
<td>-0.243* [0.113]</td>
<td>-0.123 [0.072]</td>
<td>-0.112 [0.059]</td>
<td>-0.376* [0.136]</td>
</tr>
</tbody>
</table>
DISCUSSION

- Robust “sandwich” variance estimation can be used with any of the available methods for handling dependence.
  - Hong, Chen, & Riley (2018) propose this for bivariate meta-analysis.

- Default RVE should not be used for meta-regression with predictors that vary within study.

- More attention to within- versus between-study variation in moderators.

- Improve software to make multivariate meta-analysis easier to implement.
THANKS

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REFERENCES


MORE REFERENCES


