COMBINING ROBUST VARIANCE ESTIMATION WITH MODELS FOR DEPENDENT EFFECT SIZES

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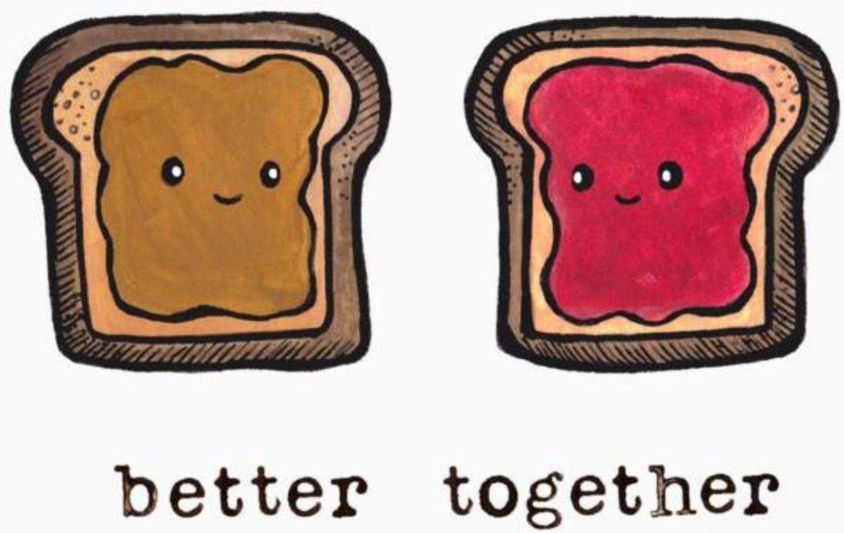
THESIS

- Many methods available for meta-analyzing dependent effect size estimates.
  - ad hoc methods (Hammering the screws)
  - model-based methods
  - robust variance estimation (RVE)

- Useful to combine RVE with model-based approaches.
  - Addresses limitations of model-based approaches.
  - Addresses limitations of default RVE implementation.

Model-based
meta-analysis
methods



Robust
variance
estimation

DEPENDENT EFFECT SIZES ARE VERY COMMON

Multiple outcomes measured on common set of participants

Treatment	O P Q
Control	O P Q

d_{O1}
d_{P1}
d_{Q1}

Outcome measured at multiple follow-up times

Treatment	O_1	O_2	O_3
Control	O_1	O_2	O_3

d_{12}
d_{22}
d_{32}

Multiple treatment conditions compared to a common control

Treatment T	O
Treatment U	O
Control	O

d_{T3}
d_{U3}

Multiple correlations from a common sample

	a	b	c	d
a				
b				
c				
d				

r_{ab}
r_{ac}
r_{ad}

FRIESE, FRANKENBACH, JOB, & LOSCHELDER (2017). DOES SELF-CONTROL TRAINING IMPROVE SELF-CONTROL: A META-ANALYSIS.

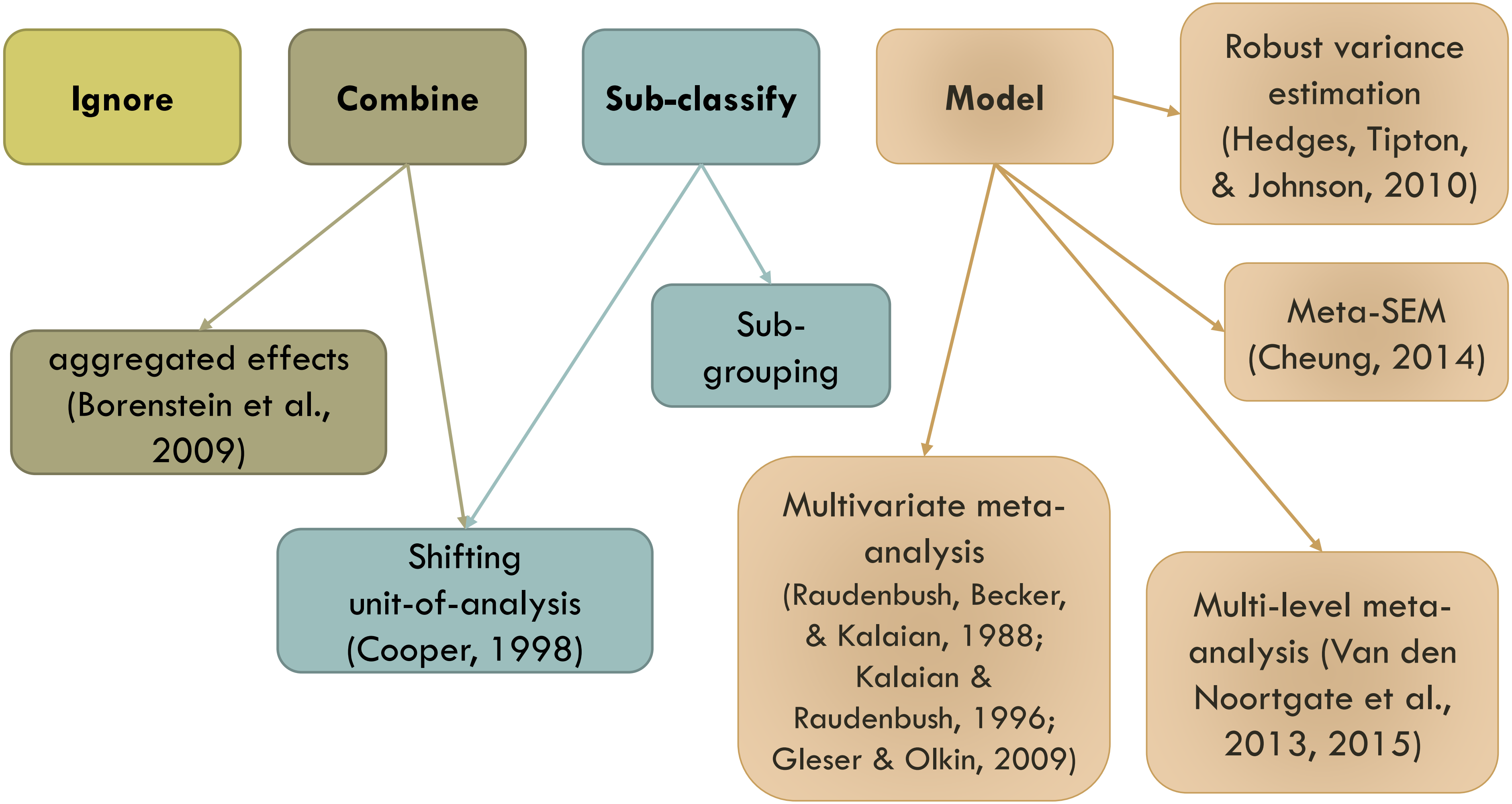
33 experimental studies, 166 effect size estimates (standardized mean differences)

- ✓ Multiple outcomes (1-13 outcomes per study, median = 2)
- ✓ Multiple follow-up times (immediate post-test and/or later follow-up)
- ✓ Multiple treatment conditions (1-4 treatment conditions per study)
- ✓ Multiple control conditions (active and/or passive control)
- ✓ 1-52 effect size estimates per study (median = 2)

COVARIANCES BETWEEN ES ESTIMATES ARE OFTEN NOT AVAILABLE

- Multiple treatments compared to common control
  - known formulas (Gleser & Olkin, 2009), easy enough to calculate
- Multiple outcomes/multiple follow-ups
  - known formulas (Gleser & Olkin, 2009)
  - require knowing correlations among outcomes/repeated measures, which are not often reported
- Multiple correlations from common sample
  - known, icky formulas (Olkin & Siotani, 1976)
  - need to know correlations between ALL variables involved

BECKER (2000) DESCRIBED FOUR BROAD STRATEGIES FOR HANDLING DEPENDENCE:



COMPARISON

Method	Requires making assumptions about ES covariances
Aggregated effects	✓
Sub-grouping	✓
Shifting unit-of-analysis	✓
Multivariate meta-analysis	✓
Multi-level meta-analysis	✓
Meta-SEM	✓
Robust variance estimation	✓ (Working model)

ROBUST VARIANCE ESTIMATION
(Hedges, Tipton, & Johnson, 2010)

- Meta-analysis/meta-regression method using "sandwich" variance estimators, which are robust to mis-specified assumptions about variance-covariance structure.

- Sandwich methods work with very general classes of models, including any of the other methods for handling dependent effects.
  - Proof: See Hedges et al. (2010, Appendix A).

COMPARISON

Method	Requires making assumptions about ES covariances	Robustness to assumptions about ES covariances
Aggregated effects	✓	Robust*
Sub-grouping	✓	Robust*
Shifting unit-of-analysis	✓	Robust*
Multivariate meta-analysis	✓	Robust*
Multi-level meta-analysis	✓	Robust*
Meta-SEM	✓	Robust*
Robust variance estimation	✓ (Working model)	Robust

* When combined with robust (sandwich) variance estimation

DEFAULT RVE IMPLEMENTATION HAS LIMITATIONS

(Hedges, Tipton, & Johnson, 2010)

- Implementation in **robumeta** packages for R and Stata.
- Limited to two "working models": correlated effects or hierarchical effects.
- Uses semi-efficient diagonal weights:

$$w_{ij} = \frac{1}{n_j\left(\bar{s}_j^2 + \hat{\tau}^2\right)}, \qquad \text{where } \bar{s}_j^2 = \frac{1}{n_j}\sum_{i=1}^{n_j} s_{ij}^2$$

- Studies contributing more effects get less weight in meta-regressions that have within-study predictors.
  - Similar to meta-regression after aggregating to the study level.

RE-ANALYSIS OF SELF-CONTROL TRAINING STUDIES

	(1) Aggregated effects	(2) Shifting unit-of-analysis	(3) Multivariate meta-analysis	(4) Multi-level meta-analysis	(5) Robust variance estimation
Overall Average ES (k = 33, N = 166)	0.281*** [0.059]		0.261*** [0.052]	0.263*** [0.054]	0.289*** [0.060]
Between-study SD	0.207		0.202	0.254	0.289
Within-study SD			0.143	0.027	
Moderator analysis by type of outcome					
Stamina (k = 16, N = 31)	0.579*** [0.157]	0.413** [0.093]	0.359*** [0.077]	0.351*** [0.071]	0.579*** [0.123]
Strength (k = 28, N = 135)	0.199** [0.071]	0.171** [0.064]	0.236*** [0.054]	0.238*** [0.055]	0.203** [0.065]
Difference	-0.380* [0.185]	-0.243* [0.113]	-0.123 [0.072]	-0.112 [0.059]	-0.376* [0.136]

DISCUSSION

- Robust "sandwich" variance estimation can be used with ***any*** of the available methods for handling dependence.
  - Hong, Chen, & Riley (2018) propose this for bivariate meta-analysis.
- Default RVE should not be used for meta-regression with predictors that vary within study.
- More attention to within- versus between-study variation in moderators.
- Improve software to make multivariate meta-analysis easier to implement.

THANKS

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