SYNTHESIS OF DEPENDENT EFFECT SIZES: VERSATILE MODELS THROUGH METAFORE & CLUBSANDWICH

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Paper: https://doi.org/10.31222/osf.io/vyfcj

January 21, 2021
Dependent effect sizes are very common

Multiple outcomes measured on common set of participants

<table>
<thead>
<tr>
<th>Treatment</th>
<th>O</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>O</td>
<td>P</td>
<td>Q</td>
</tr>
</tbody>
</table>

\[ \text{d}_{O1} \quad \text{d}_{P1} \quad \text{d}_{Q1} \]

Outcome measured at multiple follow-up times

<table>
<thead>
<tr>
<th>Treatment</th>
<th>( \text{O}_1 )</th>
<th>( \text{O}_2 )</th>
<th>( \text{O}_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>( \text{O}_1 )</td>
<td>( \text{O}_2 )</td>
<td>( \text{O}_3 )</td>
</tr>
</tbody>
</table>

\[ \text{d}_{12} \quad \text{d}_{22} \quad \text{d}_{32} \]

Multiple treatment conditions compared to a common control

<table>
<thead>
<tr>
<th>Treatment T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment U</td>
<td>O</td>
</tr>
<tr>
<td>Control</td>
<td>O</td>
</tr>
</tbody>
</table>

\[ \text{d}_{T3} \quad \text{d}_{U3} \]

Multiple correlations from a common sample

\[ \begin{array}{cccc}
  & a & b & c & d \\
 a & r_{ab} & & & \\
b & & r_{ac} & & \\
c & & & r_{ad} & \\
d & & & & \\
\end{array} \]
Robust Variance Estimation (RVE)  
(Hedges, Tipton, & Johnson, 2010)

- SEs, hypothesis tests, confidence intervals are robust to mistaken assumptions about the dependence structure of effect sizes within independent studies.
- RVE uses a “working model” to approximate the dependence structure.
  - It doesn’t have to be correct.
  - But getting closer to the true dependence structure improves precision.
Working models in robumeta

- robumeta package (Fisher, Tipton, & Hou, 2017) is the most popular implementation of RVE.
- Two available working models.

**Correlated Effects**

```
\[ \begin{align*}
  d_{O1} & \rightarrow u_1 \\
  d_{P1} & \rightarrow u_1 \\
  d_{Q1} & \rightarrow u_1 \\
  d_{O2} & \rightarrow u_2 \\
  d_{P2} & \rightarrow u_2 \\
  d_{Q2} & \rightarrow u_2 \\
  d_{P3} & \rightarrow u_3 \\
  d_{Q3} & \rightarrow u_3 \\
\end{align*} \]
```

**Hierarchical Effects**

```
\[ \begin{align*}
  d_{11} & \rightarrow v_{11} \\
  d_{21} & \rightarrow v_{21} \\
  d_{31} & \rightarrow v_{31} \\
  d_{12} & \rightarrow v_{12} \\
  d_{13} & \rightarrow v_{13} \\
  d_{23} & \rightarrow v_{23} \\
\end{align*} \]
```

\[ u_1 \rightarrow u_2 \rightarrow u_3 \]
Working models in `metafor`

- `rma.mv()` from the `metafor` package (Viechtbauer, 2010) provides a versatile set of multi-level and multi-variate models.
- These can be treated as working models, combined with RVE.

Correlated + Hierarchical Effects Model

- Allows for correlated effect size estimates.
- Allows for within-study heterogeneity in true effects.
RVE with clubSandwich

- **clubSandwich** package (Pustejovsky, 2020) provides robust standard errors, hypothesis tests, confidence intervals for many types of models.

- Supports `rma.mv()` models from `metafor`.

- Includes small-sample corrections for more accurate inference.
library(metafor)
library(clubSandwich)
TSL15 <- readRDS("Tanner-Smith-Lipsey-2015-subset.rds")

# Create a sampling variance-covariance matrix
V_mat <- impute_covariance_matrix(TSL15$V,
cluster = TSL15$studyid,
r = 0.6)

# fit working model in metafor
mod <- rma.mv(es ~ 0 + dv_cat, V = V_mat,
random = ~ 1 | studyid / esid,
data = TSL15)

# clustered SEs and CIs
conf_int(mod, vcov = "CR2")
Why metafor + clubSandwich

- Using a better approximation to the real dependence structure will give you more precise estimates of average effects/meta-regression coefficients.
- More flexible working models provide better descriptions of heterogeneity (e.g., within- and between-study variance).
- Using RVE provides protection against model misspecification.

More details, examples, code, simulation evidence: