Stabilizing measures to reconcile accuracy and equity in performance measurement

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Shrinkage estimators

Partial pooling

Borrowing information

Bayesian stabilization
## Implementation differences

<table>
<thead>
<tr>
<th></th>
<th>McCaffrey</th>
<th>Forrow</th>
<th>Gellar</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What scores to model?</strong></td>
<td>Individual-level growth scores</td>
<td>Aggregate proficiency rates</td>
<td>Average scores</td>
</tr>
<tr>
<td><strong>Where to borrow?</strong></td>
<td>Past years, across subjects <em>(only within schools)</em></td>
<td>Other schools</td>
<td>Other schools, past years, other subjects</td>
</tr>
<tr>
<td><strong>What to condition on?</strong></td>
<td>Response patterns</td>
<td>Specific groups of students</td>
<td>Subject, grade level, year</td>
</tr>
<tr>
<td><strong>How to estimate?</strong></td>
<td>Bespoke moment estimation</td>
<td>Full Bayes</td>
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</tr>
</tbody>
</table>
Implications of borrowing across schools

- Adjustments depend strongly on sample size. If borrowing across schools:

\[ \hat{\mu}_j = \bar{y}_j + \frac{1}{1 + \frac{\phi}{1-\phi} N_j} (\hat{\mu} - \bar{y}_j) \]

○ For school below the mean, reducing sample size will improve the proficiency/growth estimate.
Implications of subgroup analysis

Adjustment models within each of several specific groups:

\[ Y_{ij}^g = \mu^g + v_j^g + e_{ij}^g, \quad v_j^g \sim N(0, \tau_g^2), \quad e_{ij}^g \sim N(0, \sigma_g^2) \]

for groups \( g = 1, \ldots, G \).

- Each group gets adjusted towards a different mean, by a different factor.

As a multivariate model:

\[
\begin{pmatrix}
Y_{ij}^1 \\
Y_{ij}^2 \\
\vdots \\
Y_{ij}^G
\end{pmatrix}
= 
\begin{pmatrix}
\mu^1 \\
\mu^2 \\
\vdots \\
\mu^G
\end{pmatrix}
+ 
\begin{pmatrix}
v^1_j \\
v^2_j \\
\vdots \\
v^G_j
\end{pmatrix}
+ 
\begin{pmatrix}
e^1_{ij} \\
e^2_{ij} \\
\vdots \\
e^G_{ij}
\end{pmatrix}, 
\begin{pmatrix}
v^1_j \\
v^2_j \\
\vdots \\
v^G_j
\end{pmatrix}
\sim N(0, T), 
\begin{pmatrix}
e^1_{ij} \\
e^2_{ij} \\
\vdots \\
e^G_{ij}
\end{pmatrix}
\sim N(0, \Sigma)
\]
Model building and model checking

- Estimates based on all of these models are *model-assisted*.
  - How to develop the model for specific applications?
  - How to check that the model is suitable for purpose?

- McCaffrey: further model development seems challenging because of bespoke moment estimation framework. Would shifting to more conventional tools help in developing more stable and interpretable models?

- Forrow and Gellar: With the Bayesian approach, *posterior predictive checks* are useful general-purpose technique for evaluating model fit. But what summary quantities should the analyst check?

- All: is correlation between school size and school performance level an issue for these estimators?