

Evaluating the Transition to College Mathematics Course in Texas High Schools: Findings  
from the Second Year of Implementation

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### Abstract

Texas House Bill 5 introduced requirements that school districts partner with institutions of higher education to provide college preparatory courses in mathematics and English for high school seniors who are not yet college ready. As districts and college partners begin to respond to these provisions, there is a need for empirical research on the effects of different approaches to implementing the college preparatory courses. In response to House Bill 5 requirements, the Charles A. Dana Center has developed a model college preparatory mathematics course, Transition to College Mathematics Course (TCMC), which has been adopted by dozens of school districts across Texas over the past several school years. We examine the effects of TCMC on students' progress into post-secondary education by comparing students who participated in TCMC during the 2017-18 school year (the second year of implementation) to observationally similar students from the same cohort but who did not enroll in the course. We find that students who took TCMC graduated at higher rates than comparison students. They had similar rates of overall enrollment in post-secondary education, but enrolled in community colleges at higher rates and in 4-year colleges or universities at lower rates than did comparison students. Enrollment tended to increase over the course of four semesters after high school graduation. Relative to comparison students, students who took TCMC were also less likely to take and less likely to pass college-level math coursework. These results must be interpreted cautiously because we were unable to fully assess and account for students' college-readiness status at the start of their senior year.

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### **Introduction**

Post-secondary education has become a gateway for economic and social mobility in the United States. While both federal and state policy has sought to broaden access to higher education, there remain substantial obstacles to expanding the more crucial objective of student degree completion. One challenge is that many students exit high school under-prepared for college-level work—particularly in mathematics. Students who are not ready for college typically enroll in developmental courses in college. However, evidence indicates that some developmental coursework may actually hinder, rather than aid, students' performance in college-level coursework and progress towards graduation.

Across the country, policy-makers have pursued a range of reforms that aim to address these problems. In Texas, the state legislature responded by introducing changes to curriculum and graduation requirements in the form of Texas House Bill 5 (HB5) from the 2013 legislative session. Among its several provisions, HB5 required school districts to offer a college preparatory mathematics course for students not meeting college readiness standards in mathematics by the end of their third year of high school. This course has to be offered through a partnership with a community college or other institution of higher education. HB5 further stipulated that successful completion of the college preparatory course had to satisfy the partner institution's requirements for entry into college-level mathematics coursework, so that students would not have to later pass proficiency exams.

As a form of early college coursework, the college preparatory math course requirement introduced by HB5 could benefit students by allowing them to bypass developmental courses and immediately begin college level work. Evidence indicates that participating in early college coursework positively impacts post-secondary persistence and degree completion (An, 2013; Giani, Alexander, & Reyes, 2014; Karp, Calcagno, Hughes, Jeong, & Bailey, 2007) and might be particularly beneficial for lower-income students (An,

2013). On the other hand, the college preparatory math course targets a different sub-group of students and has somewhat different aims than typical early college coursework. Further, if high schools and community college partners were to implement the course following the pattern of a conventional developmental education course, such as a remedial Algebra II course, then the policy changes might not lead to substantive changes in instruction. In evaluating the effects of the policy change, it is therefore critical to consider the design and content of the college math preparatory course.

One curriculum designed with the goals and requirements of HB5 is the Transition to College Mathematics Course (TCMC), developed by the Charles A. Dana Center. The Dana center developed TCMC as a model college preparatory math course, melding previously developed secondary-level course materials with strategies they had used to build college-level developmental courses. TCMC differs from conventional remedial math courses in several important respects. First, the course content aligns with the multiple mathematics pathways framework (Charles A. Dana Center, 2016, 2019) adopted by many Texas higher education institutions, providing a coherent sequence of work across the transition from high school to higher education.<sup>1</sup> Second, the course involves novel material and instructional strategies, rather than repetition of content that students have already encountered. Third, the course incorporates evidence-based pedagogical approaches including putting greater emphasis on richly contextualized applications, developing students' self-regulated learning strategies and productive persistence, and varying instructional activities. Taken together, these differences provide reason to expect that student participation in TCMC could have immediate and longer-term impacts on student outcomes.

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<sup>1</sup> The TCMC materials implemented reflect the student learning outcomes contained in the HB5 College Preparatory Mathematics Content Framework, the result of a collaborative process organized by the Texas Success Center (TSC) at the Texas Association of Community Colleges (TACC). A committee consisting of content experts from both Higher Education and K-12 educators worked collaboratively to develop and revise this framework, seeking input from educators across the state. This feedback, along with the Learning Outcomes in the Lower-Division Academic Course Guide Manual (ACGM), the Texas College and Career Readiness Standards (CCRS), and other relevant materials informed the committee's work.

TCMC was offered at high schools in eight districts across central Texas during the 2016-17 school year and in thirty districts during the 2017-18 school year. In our previous report, we assessed the first-year implementation of TCMC by comparing students who participated in TCMC during the 2016-17 school year (the first year of implementation) to observationally similar students, either from a previous cohort that did not have access to TCMC or from the same cohort but who did not enroll in the course. In this first cohort, we found that students who took TCMC graduated at slightly higher rates than comparison students, but had lower rates of enrollment in post-secondary education, driven by lower rates of enrollment in 4-year colleges or universities. Enrollment differences diminished over the four semesters following graduation from high school. We also found that, relative to students in the comparison groups, students who took TCMC were less likely to pass college-level and developmental math courses. Longer-term cumulative outcomes showed larger discrepancies in rates of math course passage. However, these results must be interpreted cautiously because we were unable to fully account for students' college-readiness status at the start of their senior year.

In this report, we extend our evaluation of TCMC by examining the effects of the second year of implementation, when the course was offered in an expanded number of schools and districts throughout central Texas. During the second year, schools in seven of the eight districts from the first cohort continued to offer the course, and schools in an additional twenty three districts began implementing TCMC for the first time. Our evaluation strategy focuses on identifying a contemporaneous comparison group of students who did not enroll in TCMC, but who closely resemble students who took the course during the 2017-18 school year.<sup>2</sup> Our guiding research question remains: *Relative to taking typical high school coursework, what are the effects of participating in TCMC on high*

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<sup>2</sup> In our analysis of the first year of implementation, we also identified a previous-year comparison group. In evaluating the second year of implementation, we forgo this strategy because a substantial portion of the second cohort was implementing the course for a second year, and so the previous-year cohort included students who were already “treated” in the sense of having access to TCMC.

*school graduation, post-secondary enrollment, and progress in college-level mathematics for 12<sup>th</sup> grade students enrolled in TCMC?*

### **Methods**

In high schools that offered the course during 2017-18, enrollment in TCMC was largely at the discretion of students and school staff. The main rule guiding enrollment was that students should not be college ready at the beginning of twelfth grade. Schools could use a variety of approaches for determining college readiness, including college readiness tests, other standardized test scores, or grades. Schools did not follow any consistent approach for determining college readiness or which students should take the course, and advising practices differed from school to school. Most students who enrolled in the course were in 12<sup>th</sup> grade.

To estimate the effects of participating in the course, we constructed a comparison group of students who did not enroll in TCMC, but who are observationally similar to the group of students who did enroll in the course. We sought to create groups that are closely matched on any background characteristics that may have influenced students to enroll in the course and that may be associated with later student outcomes. If we can achieve balance on all such confounding variables, then differences between the groups in later outcomes provide estimates of the impact of enrolling in TCMC for students who chose to participate in it.

In order for this approach to provide unbiased estimates of the causal impacts of participating in the course, the set of background characteristics must include a sufficient set of confounding factors that are associated with both course enrollment and subsequent outcomes. Our analysis therefore includes prior math course-taking patterns and a standardized measure of math achievement, as well as extensive demographic information. We included prior math course-taking patterns because students who had not taken adequate classes earlier in high school and students who performed poorly in math may be

more likely to participate in TCMC in 12<sup>th</sup> grade and may also be less likely to graduate and enroll in college. One potential omitted variable is performance on Texas Success Initiative (TSI) or other standardized test that would designate students as college-ready, which might have direct effects on students' post-secondary educational choices. To the extent that TSI status at the start of students' senior year affected whether they enrolled in TCMC, above and beyond their mathematics achievement and prior math course-taking, our impact estimates may include bias from this omitted confounder. We consider the implications of this bias in the discussion section.

To identify a comparison group, we drew from the set of 12<sup>th</sup> grade students from the same school and same class year as the students who enrolled in TCMC during 2017-18, but who did not enroll in the course. Using comparison students who were contemporaries of the students who enrolled in TCMC allowed us to use consistent definitions and data sources for background characteristics and later outcome variables. The primary drawback of this approach is that the comparison students have all elected *not* to enroll in the course, and thus might differ from students who participated in TCMC in ways beyond what we can measure.<sup>3</sup> We used propensity score weighting methods (Hirano & Imbens, 2001; Schafer & Kang, 2008) to construct the comparison group and weighted least squares regression to estimate average treatment effects across the set of schools that offered TCMC for the 2017-18 school year. In the remainder of this section, we describe the implementation of TCMC during the 2017-18 school year, explain our data sources, and provide further details about our analytic strategy.

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<sup>3</sup> In our evaluation of the first year of implementation of TCMC, we also implemented another strategy, constructing a second comparison group of students from the same school, who were in 12<sup>th</sup> grade during the year *prior* to when the course was first offered. For the second year evaluation, however, several of the districts were implementing the course for the second year in a row, and so constructing a comparison group from the prior year cohort was not feasible. This strategy also had the limitation that year-to-year changes in economic conditions, state education policy, or assessment methods could produce differences in outcomes between students who enrolled in TCMC and comparable students from the prior year.

## **TCMC Implementation**

The Dana Center worked with higher education partners to recruit high schools interested in offering the course and participating in evaluation activities. Thirty districts agreed to participate for the 2017-18 school year, of which seven had participated in the first year of implementation. To satisfy the college readiness course requirements of HB5, participating districts partnered with several different community colleges, including Austin Community College, Lone Star College, and Lee College. Participating high schools received free curriculum materials and professional development, as well as ongoing technical support. Professional development consisted of a two-day, in-person summer training a further, one-day training during winter break. In return, participating schools agreed to assist with evaluation activities by administering surveys to and providing further administrative data on students enrolled in the course.

## **Data Sources**

We used statewide longitudinal data collected by the Texas Education Agency (TEA), and Texas Higher Education Coordinating Board (THECB). We accessed the data through the Education Research Center at The University of Texas at Austin. The data include student demographics, course enrollment and completion in elementary and secondary schools, performance on State of Texas Assessments of Academic Readiness (STAAR) assessments, and high school graduation. The data also include information on student matriculation into in-state college.

**Analytic Samples.** Using the TEA course completion data, we identified students enrolled in classes labeled as College Preparatory Mathematics or Independent Study in Math in participating districts. To identify sections of TCMC, we compared class name, class identification, class period, teacher identification number, and the number of sections to separate evaluation data provided to us by the Dana Center. To define the treatment group, we identified 12<sup>th</sup> grade students enrolled in sections of TCMC in the thirty focal

districts. We excluded a small number of students who appeared to be enrolled in sections of TCMC but were not actually enrolled in the focal campuses according to the TEA attendance data. To define the comparison group, we identified 12<sup>th</sup> grade students who were enrolled at the focal campuses during the 2017-18 school year but not enrolled in TCMC. For students who were enrolled at multiple campuses during a single year (i.e., because they switched schools mid-year), we retained the record from the school with the maximum number of eligible days.

**Outcomes.** Just as in our previous evaluation, we assessed the impacts of TCMC on outcomes related to students' post-secondary success. The primary outcomes of interest are enrollment and passage rates for college-level math courses. To provide a more complete picture, we also examined enrollment and passage rates for developmental math courses at the post-secondary level. Graduation from high school and enrollment in post-secondary education are necessary steps to begin college-level math courses. Therefore, we also examined impacts on high school graduation and college enrollment rates. Because the design of college preparatory course policies incentivize enrollment at partner community colleges, we dis-aggregated college enrollment rates by sector, distinguishing between community college, public four-year colleges or universities, and private four-year colleges. We examined high school graduation rates by the end of the students' 12<sup>th</sup> grade year and we also examined cumulative graduation rates within two years after the students began TCMC. For all of the college-level outcomes, we examined cumulative rates for four semesters after the students' high school year (Fall, Spring, Summer, Fall).

Table 1 provides definitions of the outcomes and lists the data sources from which they were obtained. We obtained data on high school graduation from TEA graduation datasets. We obtained college enrollment and course-taking data from THECB enrollment and course datasets. Post-secondary outcomes were therefore limited to students who enrolled in institutions of higher education within the state of Texas, as recorded in THECB data. We excluded non-degree seeking students and dual-credit students.

Table 1  
*Outcome definitions and sources*

Outcome	Definition	Source
High School Graduation	Graduating high school.	p_graduate
Post-Secondary Enrollment	Enrollment in community college, public and private four year-institutions, or health programs.	cbm_001
Post-Secondary Enrollment: Public Four Year	Enrollment in public four-year colleges.	cbm_001
Post-Secondary Enrollment: Private Four Year	Enrollment in private four-year colleges.	cbm_001
Post-Secondary Enrollment: Community	Enrollment in community colleges.	cbm_001
Post-Secondary Enrollment: Partner	Enrollment in community colleges that partnered with the focal districts to offer TCMC.	cbm_001
Post-Secondary Math Enrollment: College-Level	Enrollment in non-developmental college math courses. For all the outcomes below based on cbm_00s, we excluded students who took courses for dual credit. We excluded lab, co-op, internship, clinical and practicum courses.	cbm_00s
Post-Secondary Math Enrollment: Developmental	Enrollment in developmental college math courses.	cbm_00s
Post-Secondary Math Passing: College-Level	Passing non-developmental college math courses. For duplicated records, we kept the passing grade.	cbm_00s
Post-Secondary Math Passing: Developmental	Passing developmental college math courses. For duplicated records, we kept the passing grade.	cbm_00s

**Covariates.** As in our evaluation of the first year of implementation, we used an extensive set of background characteristics to develop propensity score weights and estimate impacts. Our main covariates included current demographic status and history, program and service enrollment history, prior math course enrollments, prior math performance, and current enrollment in Advanced Placement math or dual-credit math courses. Because TCMC is designed for students who are under-prepared for college-level math courses, it is highly likely that prior math course-taking and achievement influenced whether students were advised or required to take TCMC, and prior math achievement is also clearly related to later student outcomes. We also account for current enrollment in Advanced Placement math courses or dual-credit math courses because enrollment in these courses is generally indicative of college readiness, and so students who enroll in them would be very unlikely to enroll in TCMC.<sup>4</sup> With respect to student socio-economic background, Michelmore and Dynarski (2017) demonstrated that the effects of economic disadvantage on educational outcomes can be better captured by using longitudinal measures of income that estimate duration of disadvantage than by using a single point-in-time measure. We therefore included longitudinal measures of economic disadvantage as well as immigrant status, history of special education enrollment, gifted enrollment, and drop-out at-risk status.

We assembled the following covariates:

- (1) Current demographic and program enrollment status: Sex, race/ethnicity categories, immigrant status, economic disadvantage status<sup>5</sup>, gifted program enrollment, special education program enrollment, and dropout at-risk status. These variables were drawn from the attendance and enrollment data from the 12<sup>th</sup> grade year.

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<sup>4</sup> Indicators for Advanced Placement math courses and dual-credit math courses were not used in our analysis of the first year of implementation. We included them here to better identify the set of comparison students who were not college ready, and thus could have chosen to enroll in TCMC.

<sup>5</sup> Economic disadvantage categories include: eligibility for free lunch as part of the National School Lunch And Child Nutrition Program (NSLP), eligibility for reduced-price lunch under NSLP, and other economic disadvantage. Other economic disadvantage is determined from sources other than NSLP eligibility, including having annual income below the federal poverty line or being eligible for public assistance such as through Temporary Assistance to Needy Families.

- (2) Demographic and program enrollment history: The number of years of available data; the number of years that a student was indicated as being in any of the categories for economic disadvantage and the number of years that the student was indicated as being in special education program, in gifted program, an immigrant, and at risk for dropping out; the proportion of years (the number of years the student was in the category divided by the number of years of record available) for the economic disadvantage categories; and whether the student was ever indicated as enrolled in a special education program, enrolled in a gifted program, an immigrant, and at risk.
- (3) Math course-taking history: whether students took (and passed or failed or did not complete) 8th grade math (four years prior), Algebra I (three, four, and five years prior), Geometry (one, two, and three years prior), Algebra II (one, two, and three years prior), Precalculus (one and two years prior), Algebraic Reasoning (one and two years prior), and Mathematical Models with Applications (one, two, and three years prior).
- (4) STAAR scores: Score on Algebra I end-of-course exam. We retained the earliest score if a student took the test multiple times.
- (5) Advanced Placement and dual-credit course-taking: whether students enrolled in an Advanced Placement math course or a dual-credit math course during their senior year.

Full definitions of these variables and data sources from which they were obtained are in Table 2. For categorical variables with missing data, we created an additional category indicating missingness. For continuous variables, missing data were imputed with the mean of the variable in the TCMC group within the given high school. For the continuous variables, we also created additional variables indicating missing values (Rosenbaum & Rubin, 1984).

Table 2  
*Covariate definitions and sources*

Variable	Definition	Source
Sex	Sex- male/female.	p_attend_demog
Race/Ethnicity	Race/ethnicity- Asian American, African American, Hispanic, American Indian, Pacific Islander, Multiracial, and White.	p_attend_demog
Economic disadvantage	Indicates economic disadvantage status: free lunch status, reduced lunch status, no disadvantage or other disadvantage. We dummy coded the variable and took the average of the data from the attend and enroll datasets for the 12th grade year.	p_attend_demog & p_enroll_demog
At risk for dropping out	Indicates whether a student was at risk for dropping out of school according to state-defined criteria as of the beginning of the 12th grade year .	p_enroll_demog
Giftedness	Indicates whether a student participated in state-approved gifted and talented program. We took the average of the data from the attend and enroll datasets for the 12th grade year.	p_attend_demog & p_enroll_demog
Immigrant status	Indicates whether a student was identified as an immigrant according to the definition in Title III of No Child Left Behind Act of 2001- individuals who are aged 3 through 21, were not born in any state, and have not been attending one or more schools in any one or more states for more than three full academic years. The data is from the beginning of the 12th grade year.	p_enroll_demog
Special education status	Indicates whether a student participated in special education instructional and related services program or general education program using special education services, supplementary aids, or other special arrangements. We took the average of the data from the attend and enroll datasets for the 12th grade year.	p_attend_demog & p_enroll_demog
Limited English proficiency	Indicates whether a student was limited English proficient as determined by Language Proficiency Assessment Committee (LPAC) as of the end of the 12th grade year.	p_attend_demog
Prior math course-taking	Indicates whether a student took Grade 8 Mathematics (four years prior), Algebra I (three, four, or five years prior), Geometry (one, two, or three years prior), Algebra II (one, two, or three years prior) Pre-calculus (one or two years prior), Algebraic Reasoning (one or two years prior), and Mathematical Models with Applications (one, two, or three years prior). Variables for these courses indicated if the student passed, failed, or did not take the class in the given year.	p_course_complete
Prior math performance	STAAR end-of-course exam score for Algebra I, completed between 2014 and 2017. For duplicate scores (i.e., if the students took the test in multiple years), we kept the earliest score.	staareoca1
AP and DC enrollment	Indicates whether a student took an Advanced Placement math course or a dual-credit math course during their senior year.	p_course_complete
History of economic disadvantage, at-risk for dropping out, giftedness, immigrant status, and special education status	We tracked these variables from 2010 through 2017. Variables include (1) the number of years of available tracked data; (2) the number of years that a student was indicated as being in any of the categories for economic disadvantage and the number of years that the student was indicated as being in special education program, in gifted program, an immigrant, and at risk; (3) the proportion of years (the number of years the student was in the category divided by the number of years of record available) for the economic disadvantages categories; and, (4) if the student was ever indicated as being in special education program, in gifted program, an immigrant, and at risk.	p_enroll_demog

Tables 3 through 5 show the distribution of the covariates in the TCMC and the two comparison groups. The TCMC group had higher percentages of African American and Hispanic students, while the comparison group had higher percentages of White and Asian students. Relative to the comparison group, the TCMC group had a higher percentage of students currently receiving free lunch, higher percentages of students who were at risk for dropping out and were ever at risk, lower percentages of students who were currently in gifted programs and ever in gifted programs, and lower percentages of students currently in special education programs and ever in these programs. The TCMC group also had higher average number and proportion of years of receiving free lunch and having other disadvantage, lower average number and proportion of years of receiving reduced-price lunch and being not economically disadvantaged, higher average number of years of being at risk for graduation, and lower average number of years of being in gifted programs and being in special education programs.

Table 3  
*Distribution of covariates: Demographics*

Variable	Comparison	TCMC
<b>Sample Size</b>		
N	20854	2456
<b>Sex</b>		
Female	49%	51%
Male	51%	49%
<b>Race or Ethnicity</b>		
Asian American	4%	<=1%
African American	10%	16%
Hispanic	60%	63%
American Indian	<=1%	<=1%
Pacific Islander	<=1%	<=1%
Multiracial	<=2%	2%
White	24%	18%
<b>Economic Disadvantage</b>		
Free Lunch	44%	51%
Reduced Lunch	45%	39%
Not Disadvantaged	4%	5%
Other Disadvantage	5%	6%
<b>At Risk</b>		
At Risk	47%	64%
At Risk Ever	68%	83%
<b>Giftedness</b>		
Giftedness	8%	0.02
Giftedness Ever	11%	<=5%
<b>Immigrant</b>		
Immigrant	2%	<=1%
Immigrant Ever	5%	<=6%
<b>Limited English Proficiency</b>		
Limited English Proficiency	8%	10%
<b>Special Education</b>		
Special Education	10%	4%
Special Education Ever	12%	<=8%
<b>AP/Dual-Credit</b>		
AP math enrollment	17%	<=1%
Dual-credit math enrollment	5%	<=1%
<b>Missingness</b>		
Algebra 1 STAAR Missing	17%	10%
At Risk Missing	2%	<=1%
At Risk Ever Missing	2%	<=1%
Giftedness Ever Missing	2%	<=1%
Immigrant Missing	2%	<=1%
Immigrant Ever Missing	2%	<=1%
Special Education Missing	2%	<=1%
Tracking Missing	2%	<=1%

Table 4

*Distribution of covariates: Demographic history and academic performance*

Variable	TCMC		Comparison	
	M	SD	M	SD
Demographic Tracking Number of Years	7.22	1.85	7.36	1.66
Proportion of Years with Missing Enrollment Data	0.02	0.13	0.00	0.04
<b>Economic Disadvantage History</b>				
Free Proportion of Years	0.46	0.40	0.52	0.39
Reduced Proportion of Years	0.08	0.16	0.08	0.16
Other Disadvantage Proportion of Years	0.04	0.12	0.06	0.16
<b>At Risk History</b>				
At Risk Number of Years	3.11	2.93	4.10	2.86
<b>Giftedness History</b>				
Gifted Number of Years	0.66	2.02	0.19	1.03
<b>Immigrant History</b>				
Immigrant Number of Years	0.12	0.56	0.11	0.49
<b>Special Education History</b>				
Special Education Number of Years	0.75	2.17	0.37	1.51
<b>STAAR Scores</b>				
Algebra 1 End of Course STAAR Scores	4009.53	430.90	3828.58	290.71

In terms of academics, the TCMC group had lower average Algebra I STAAR scores than the comparison group (Table 4). A higher percentage of the comparison group began taking high school math credits (i.e., Algebra I) in 8th grade, four years prior to the start of senior year. This pattern also held for courses taken later, in that students in the comparison group were more likely to have taken Algebra I, Geometry, Algebra II and Precalculus a year prior to when they would normally be required to take the courses (Table 5). The comparison group also had higher passing rates for these courses. For students who took Algebra I, Geometry and Algebra II in the year required, those in the TCMC group had higher passing rates than those in the comparison group. Compared to the TCMC group, a larger fraction of students in the comparison group took Precalculus or Mathematical Models with Applications during their Junior year. Notably, 17% of the comparison group enrolled in an Advanced Placement math course during their senior year and 5% enrolled in a dual-credit math course, whereas hardly any TCMC students enrolled in Advanced Placement or dual-credit math courses.

Table 5  
*Distribution of covariates: Course-taking history*

Course	Group	Pass	Fail	Incomplete	Did not take	Missing
<b>Math Grade 8</b>						
Math Grade 8 4 Yrs	TCMC	9%	<1%	<1%	84%	<7%
	Comparison	<6%	<1%	<1%	85%	9%
Math Grade 8 A 4 Yrs	TCMC	70%	<3%	<1%	21%	6%
	Comparison	52%	<2%	<1%	37%	9%
<b>Algebra I</b>						
Algebra I 5 Yrs	TCMC	<1%	<1%	<1%	91%	<9%
	Comparison	<1%	<1%	<1%	88%	10%
Algebra I 4 Yrs	TCMC	<6%	<1%	<1%	87%	6%
	Comparison	21%	<1%	<1%	69%	<10%
Algebra I 3 Yrs	TCMC	79%	8%	<1%	9%	<4%
	Comparison	55%	7%	<1%	32%	<5%
<b>Algebra II</b>						
Algebra II 3 Yrs	TCMC	<1%	<1%	<1%	96%	<4%
	Comparison	<2%	<1%	<1%	93%	5%
Algebra II 2 Yrs	TCMC	6%	<2%	<1%	90%	2%
	Comparison	23%	<2%	<1%	71%	3%
Algebra II 1 Yr	TCMC	71%	15%	<1%	<14%	<1%
	Comparison	40%	7%	<1%	51%	<2%
<b>Algebraic Reasoning</b>						
Algebraic Reasoning 2 Yrs	TCMC	<1%	<1%	<1%	98%	<2%
	Comparison	<1%	<1%	<1%	97%	<3%
Algebraic Reasoning 1 Yr	TCMC	<2%	<1%	<1%	98%	<1%
	Comparison	6%	<1%	<1%	92%	2%
<b>Geometry</b>						
Geometry 3 Yrs	TCMC	6%	<1%	<1%	89%	<4%
	Comparison	21%	<2%	<1%	71%	5%
Geometry 2 Yrs	TCMC	76%	12%	<1%	10%	<2%
	Comparison	52%	10%	<1%	35%	<3%
Geometry 1 Yr	TCMC	4%	<1%	<1%	94%	<1%
	Comparison	6%	2%	<1%	91%	<2%
<b>Mathematical Models</b>						
Mathematical Models 3 Yrs	TCMC	<1%	<1%	<1%	95%	4%
	Comparison	<1%	<1%	<1%	93%	5%
Mathematical Models 2 Yrs	TCMC	2%	<1%	<1%	96%	<2%
	Comparison	<4%	<1%	<1%	93%	3%
Mathematical Models 1 Yr	TCMC	3%	<1%	<1%	96%	<1%
	Comparison	10%	<1%	<1%	87%	2%
Precalculus 2 Yrs	TCMC	<1%	<1%	<1%	98%	<2%
	Comparison	<1%	<1%	<1%	96%	3%
Precalculus 1 Yr	TCMC	4%	<2%	<1%	93%	<1%
	Comparison	20%	<1%	<1%	77%	2%

*Note:* For two-semester courses, percentages reflect performance in second semester. 8th Grade Math A refers to a year long non-high school course. Yrs or Yr indicates the number of years before 12th grade when the students took the course.

## Analytic Models

Our analytic approach is identical to the methods used in our previous evaluation. Specifically, we used propensity score analysis and weighted outcome regression to estimate the average causal effect of participating in TCMC compared to taking typical high school coursework. All analyses were conducted in R (version 3.3.1; R Core Team, 2016).

To the construct comparison group, we used a recently developed algorithm called the generalized boosted regression model (GBRM; McCaffrey, Ridgeway, & Morral, 2004), an extension of propensity score methods introduced by Rosenbaum and Rubin (1983). GBRM differs from standard methods (such as logistic regression) in that it is a non-parametric model, which does not impose strong assumptions about the functional form of the relationship between the propensity score and the covariates. Furthermore, rather than estimating the model by optimizing predictive fit, GBRM directly optimizes a measure of comparability between treated and untreated units. Thus, GBRM is particularly well-suited for estimating propensity scores based on a large set of covariates, as we use here (Lee, Lessler, & Stuart, 2009; McCaffrey et al., 2004).

We estimated propensity scores via GBRM with the *twang* package (version 1.4-9.5; Ridgeway, McCaffrey, Morral, Griffin, & Burgette, 2016) with default settings for the number of trees, interaction depth, and shrinkage. The propensity score model included all of the covariates listed in Table 2. We included school and district IDs as additional covariates in the propensity score model, which has the effect of allowing the GBRM algorithm to discover school-by-covariate interactions that improve balance.

Based on the propensity score model estimated using GBRM, average treatment effect for the treated (ATT) weights were calculated as

$$w_{ij} = D_{ij} + (1 - D_{ij}) \left( \frac{\hat{p}_{ij}}{1 - \hat{p}_{ij}} \right), \quad (1)$$

for student  $i = 1, \dots, n_j$  in school  $j$ , where  $D_{ij}$  is an indicator term equal to one if the

student was enrolled in TCMC, and  $\hat{p}_{ij}$  is the estimated propensity score for the student. Weights were standardized to sum to one within the TCMC and comparison groups.

To estimate the average effect of enrolling in TCMC for students who participated in the course, we regressed each outcome on the covariates, indicators for each school, covariate-by-treatment interactions, and school-by-treatment interactions. We used the following analytic model:

$$Y_{ij} = \alpha_j + \beta_j D_{ij} + \gamma X_{ij} + \delta X_{ij} D_{ij} + e_{ij} \quad (2)$$

Here,  $Y_{ij}$  is the outcome of student  $i$  in school  $j$ ,  $\alpha_j$  is an indicator for the school in which the student is enrolled,  $D_{ij}$  is an indicator equal to one if the student was enrolled in TCMC and equal to zero if the student was in the comparison group, and  $X_{ij}$  is a set of covariates encoding student background characteristics. The set of covariates included all the variables listed in Table 2. Treatment was allowed to interact with each of the covariates, with  $\delta$  representing the vector of interactions. Treatment was also allowed to interact with school, thereby allowing that the effects of participating in TCMC could vary by school. Categorical covariates were dummy coded. Each covariate was centered at its unweighted mean in the treated group within each school. Because the covariates were centered in this way,  $\beta_j$  term represents the school-specific ATT and  $\alpha_j$  represents the expected school-specific average outcome if the students in the TCMC group had not taken the course.

To estimate an overall average effect for students who took TCMC ( $\beta$ ), we calculated a weighted average of the school-specific estimates, with weights based on the size of the TCMC group in each school. Let  $N_{1j}$  denote the number of students enrolled in TCMC in school  $j$  and  $N_1$  denote the total number of students enrolled in TCMC across schools. We

calculated the overall average treatment effect estimate as:

$$\hat{\beta} = \sum_{j=1}^J \left( \frac{N_{1j}}{N_1} \right) \hat{\beta}_j, \quad (3)$$

where  $\hat{\beta}_j$  is the school-specific estimate. To calculate the standard error of the overall estimate, we first calculated standard errors for the school-specific estimates using HC0-type standard errors (Zeileis, 2004, version 2.3–4), which are robust to heteroskedasticity in the regression errors of Equation (2). Let  $V_j$  be the estimated sampling variance of  $\beta_j$ . The variance of the overall average treatment effect was then calculated as:

$$V^{\beta} = \sum_{j=1}^J \left( \frac{N_{1j}}{N_1} \right)^2 V_j. \quad (4)$$

We conducted hypothesis tests and calculated 95% confidence intervals for the average effect based on large-sample normal approximations.

## Results

### Propensity Score Distribution: Common Support

Figure 1 shows the distribution of logit propensity scores for the TCMC and comparison groups. We can see that there are TCMC students (in blue) with very high propensities, which are higher than the maximum propensity among comparison students. Students with such high propensity scores do not have comparable corresponding students in the comparison group. However, such students comprise only a very small fraction of the full TCMC group, and so we retain them in the analysis.

Table 6  
*Sample sizes for contemporaneous comparison group*

Quantity	TCMC	Comparison
Sample size (unweighted)	2456	20854
Effective sample size (weighted)	2456	7971

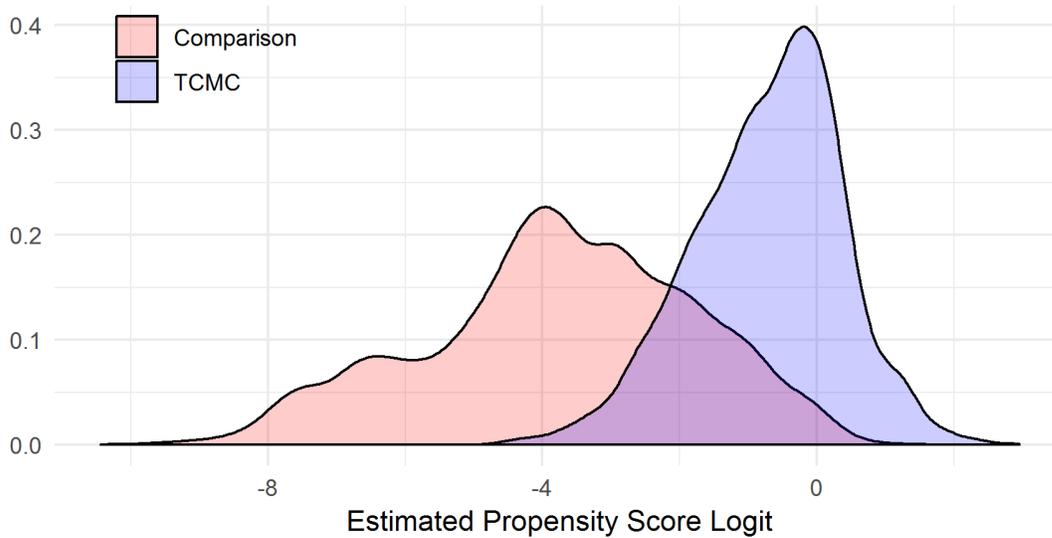


Figure 1. Propensity score support

Table 6 reports sample sizes and effective sample sizes for the TCMC group and contemporaneous comparison group. In the table, the effective sample size corresponds to the number of observations from an unweighted sample that would yield the same level of precision as a weighted sample (Ridgeway et al., 2016). Prior to re-weighting, the comparison group is comprised of over 20,000 students. After re-weighting, the effective sample size is reduced to under 10,000 students, but is still several times larger than the sample size of the TCMC group and sufficient for obtaining precise impact estimates.

### Covariate Balance

We assessed balance on all of the covariates that were included in the propensity score model. Figures 2, 3, and 4 depict the results of the balance assessment prior to and after re-weighting the comparison group. For continuous covariates, standardized mean differences were calculated. For binary covariates, raw differences in proportions were calculated. Standardized mean differences and differences in proportions were calculated prior to (Unadjusted) and after (Adjusted) propensity score weighting. The dashed line in the figures below represent threshold values of  $-0.1$  and  $0.1$ , as recommended by Stuart (2008); mean differences within the dashed lines indicate acceptable levels of imbalance.

After re-weighting based on propensity scores, mean differences on the covariates were close to zero for all the included covariates. In particular, 2 illustrates that the re-weighting process led to a comparison group that excluded students who enrolled in Advanced Placement or dual-credit math courses during or prior to their senior year, so that the re-weighted comparison group is very well balanced with the TCMC group. Overall, the balance results indicate that the set of students in the comparison group is very similar to the set of students who took TCMC in terms of demographic characteristics, academic performance, and past math coursework. To the extent that these background characteristics explain whether a student enrolled in TCMC, differences in outcomes between students who took TCMC and students in the comparison group can be attributed to the impact of the program.

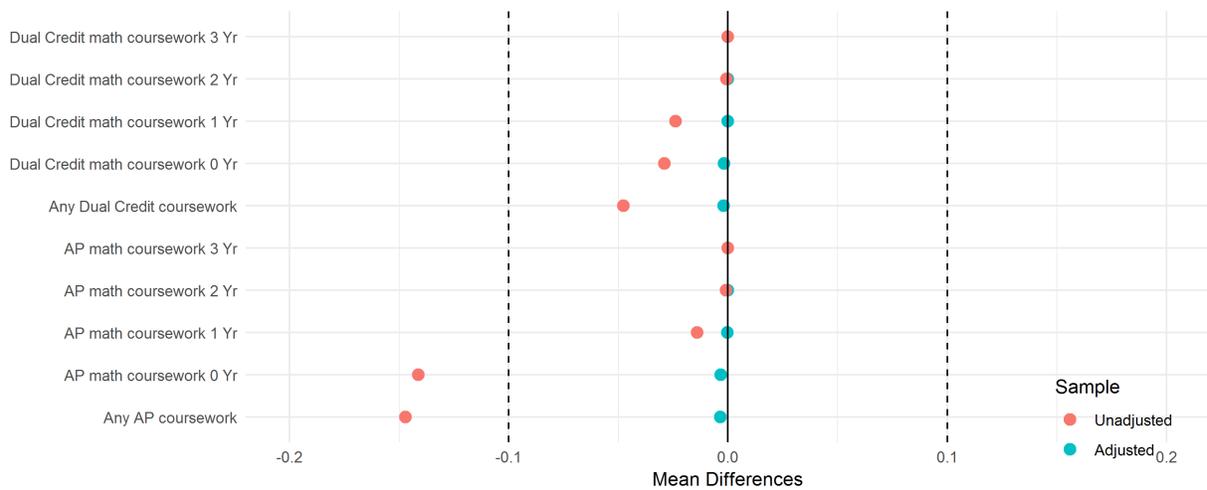


Figure 2. Balance results: Advanced placement and dual-credit course-taking

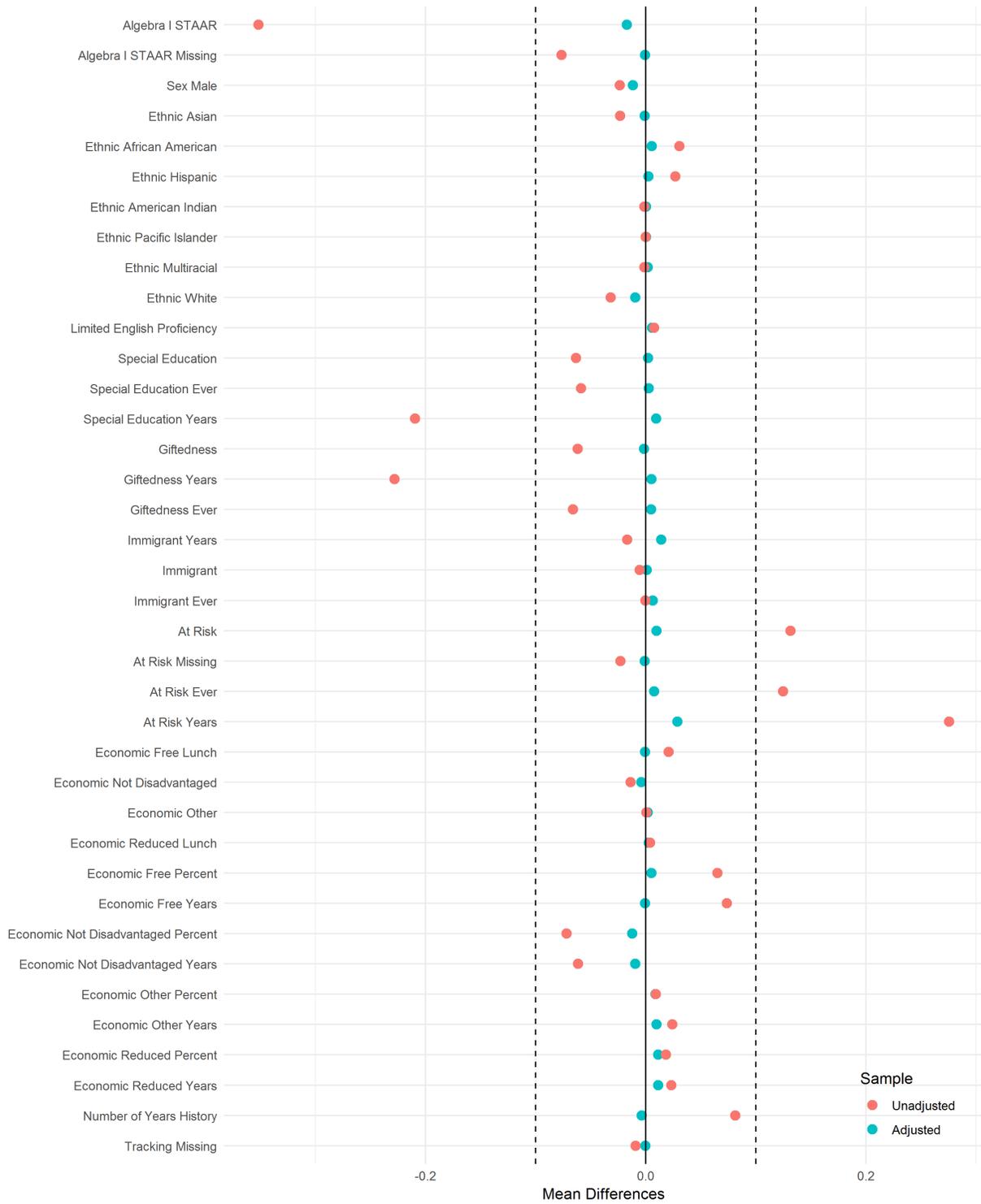


Figure 3. Balance results: Demographics and academic measures

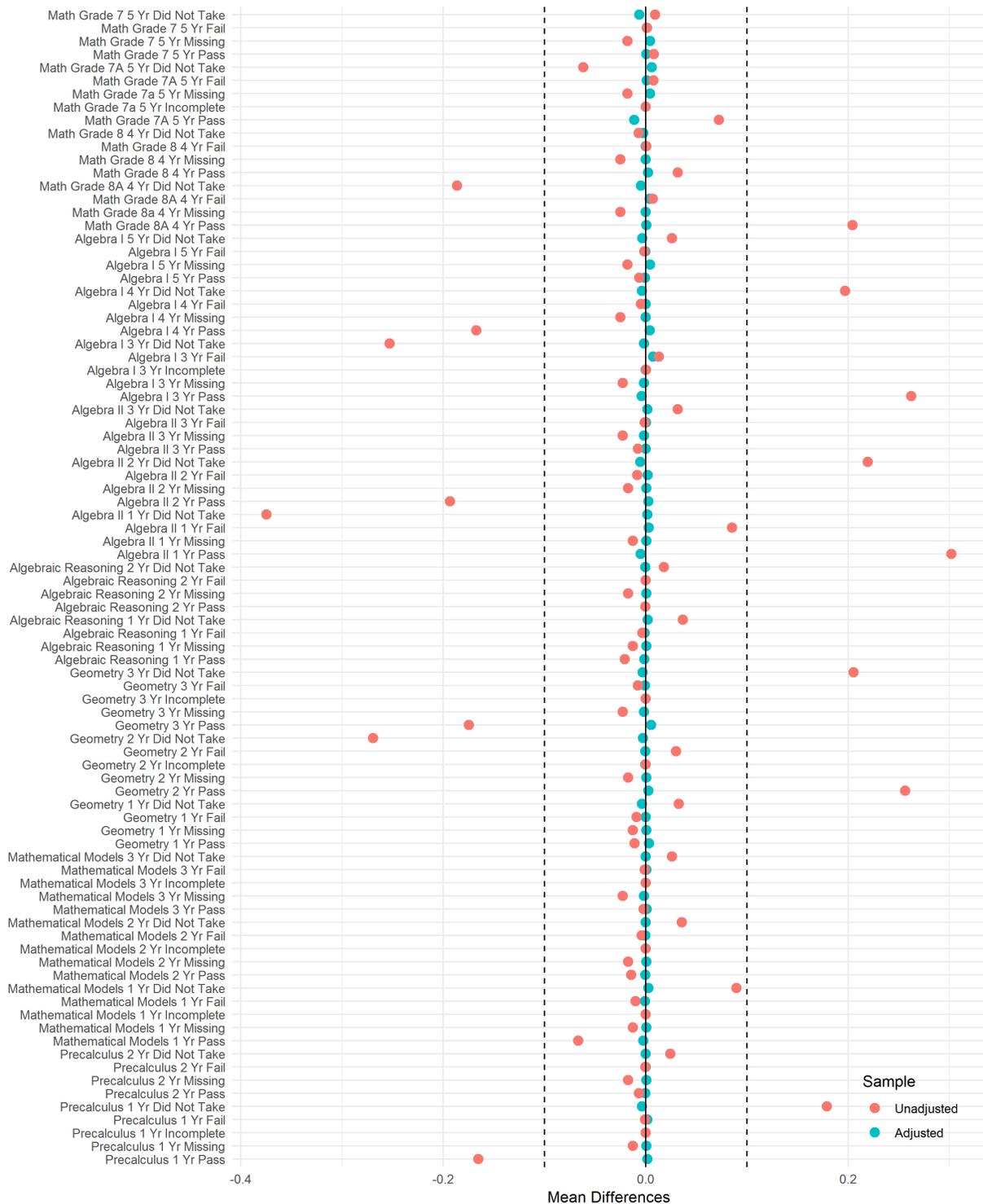


Figure 4. Balance results: Course-taking patterns

## Impact Estimates

Figures 5 and 6 and Tables 7 and 8 report impact estimates for the full set of outcomes that we examined. Figure 5 depicts the estimated effects of taking TCMC, averaged across schools, along with the 95% confidence interval bands, for high school graduation and post-secondary enrollment outcomes. The dashed line on zero indicates no effect; interval bands that cross the dashed line represent estimates that are not statistically distinguishable from zero. Table 7 presents estimated rates of the graduation and college enrollment outcomes for the TCMC and the re-weighted comparison group. The rates for the comparison group can be interpreted as the baseline rates of the outcomes (i.e., if TCMC had not been available). The column labeled “Difference” is the estimated difference between the TCMC group and the comparison group, i.e., our estimate of the impact of taking TCMC. Table 7 also reports standard errors (SE) and p-values (p) associated with the impact estimates. We discuss significance results based on a conventional  $\alpha$  value of 0.05.

We found that students enrolled in TCMC were more likely to graduate on time compared to students in the comparison group, by 4.7 percentage points, 95% CI [3.9, 5.5]. Looking across post-secondary institution types, students who enrolled in TCMC were slightly more likely to enroll in post-secondary education during the semesters following graduation, although the differences are not statistically distinguishable from zero. During the Fall after high school graduation, 36.4% of TCMC students and 35.8% of comparison students enrolled in some form of college or university, a difference of 0.6 percentage points, 95% CI [-1.8, 3.0]. Over later semesters, enrollment grew among both groups and a difference in enrollment rates appeared, widening to 2.0 percentage points, 95% CI [-0.4, 4.4]. However, because the estimates are not statistically distinguishable from zero, we cannot rule out the possibility that students in both groups enrolled at equivalent rates.

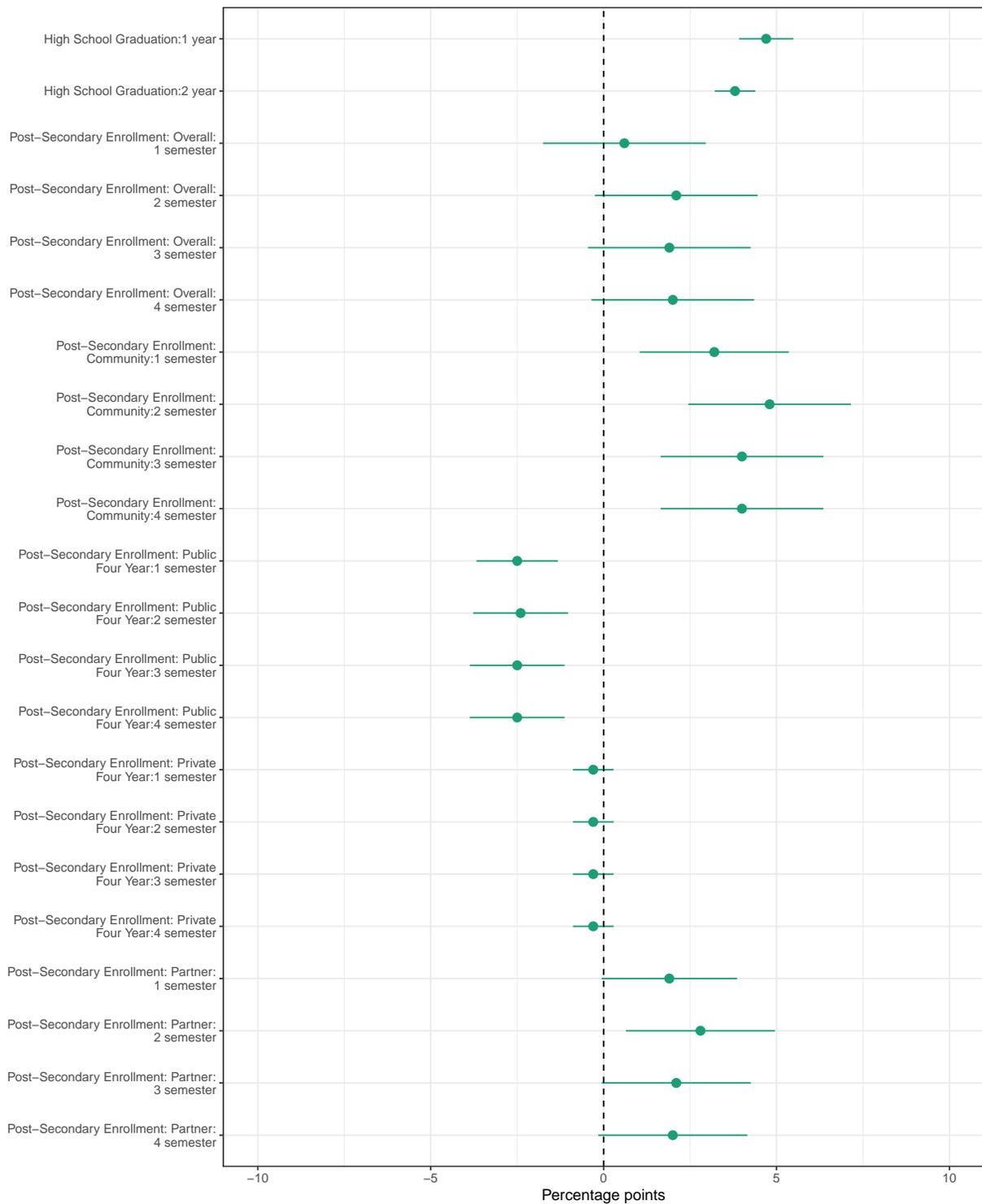


Figure 5. Overall average effects of taking TCMC on high school graduation and post-secondary enrollment rates. Dots represent point estimates. Lines correspond to 95% confidence intervals.

Table 7

*Estimated average effects of TCMC on high school graduation and post-secondary enrollment rates*

Outcome	TCMC	Comparison	Difference	SE	p
<b>High School Graduation</b>					
1 year	98.7	94.0	4.7	0.4	<0.001
2 year	99.2	95.4	3.8	0.3	<0.001
<b>Post-Secondary Enrollment: Overall</b>					
1 semester	36.4	35.8	0.6	1.2	0.603
2 semester	41.9	39.8	2.1	1.2	0.074
3 semester	42.5	40.6	1.9	1.2	0.105
4 semester	42.8	40.8	2.0	1.2	0.093
<b>Post-Secondary Enrollment: Community</b>					
1 semester	27.9	24.6	3.2	1.1	0.003
2 semester	33.3	28.5	4.8	1.2	<0.001
3 semester	34.9	30.9	4.0	1.2	0.001
4 semester	34.9	30.9	4.0	1.2	0.001
<b>Post-Secondary Enrollment: Public Four Year</b>					
1 semester	7.2	9.7	-2.5	0.6	<0.001
2 semester	7.7	10.1	-2.4	0.7	<0.001
3 semester	7.7	10.2	-2.5	0.7	<0.001
4 semester	8.8	11.3	-2.5	0.7	<0.001
<b>Post-Secondary Enrollment: Private Four Year</b>					
1 semester	1.5	1.8	-0.3	0.3	0.417
2 semester	1.5	1.8	-0.3	0.3	0.391
3 semester	1.5	1.8	-0.3	0.3	0.391
4 semester	1.7	2.0	-0.3	0.3	0.367
<b>Post-Secondary Enrollment: Partner</b>					
1 semester	21.3	19.4	1.9	1.0	0.055
2 semester	25.6	22.7	2.8	1.1	0.007
3 semester	26.9	24.8	2.1	1.1	0.049
4 semester	26.9	24.8	2.0	1.1	0.059

Looking only at overall enrollment rates does not provide a full picture of the differences between TCMC and comparison group students, however. Although overall effects on enrollment were small, it appears that students who enrolled in TCMC differed from comparison students in terms of *where* they enrolled, with more TCMC students enrolling in community colleges and fewer enrolling in four-year institutions. Relative to students in the comparison group, TCMC students were more likely to subsequently enroll in community college, with 27.9% of TCMC students and 24.6% of comparison students

enrolling for the Fall semester after high school graduation. Differences in community college enrollment rates were statistically distinguishable from zero and grew to 4.0 percentage points, 95% CI [1.6, 6.4] by the Fall semester one year after high school graduation. We observed similar patterns and trends when looking at enrollment rates for community colleges who were district partners.

The differences in community college enrollment rates among TCMC students are partially offset by lower rates of enrollment in four-year colleges and universities. In the Fall semester after high school graduation, 7.2% of TCMC students and 9.7% of comparison students enrolled in public, four-year institutions, a difference of -2.5 percentage points, 95% CI [-3.7, -1.3]. This difference remained stable in subsequent semesters, although enrollment in four-year institutions grew slightly among both groups.

Table 8

*Estimated average effects of TCMC on post-secondary math course-taking and course-passing rates*

Outcome	TCMC	Comparison	Difference	SE	p
<b>Post-Secondary Math Enrollment: College-Level</b>					
1 semester	4.4	6.0	-1.7	0.5	0.001
2 semester	11.6	14.0	-2.4	0.8	0.004
3 semester	12.3	14.8	-2.4	0.8	0.003
4 semester	12.3	14.8	-2.4	0.8	0.003
<b>Post-Secondary Math Enrollment: Developmental</b>					
1 semester	1.1	1.2	-0.2	0.3	0.523
2 semester	3.4	3.3	0.1	0.5	0.775
3 semester	3.5	3.4	0.2	0.5	0.695
4 semester	3.5	3.4	0.2	0.5	0.695
<b>Post-Secondary Math Passing: College-Level</b>					
1 semester	3.0	4.5	-1.6	0.4	<0.001
2 semester	6.9	9.3	-2.4	0.6	<0.001
3 semester	7.6	10.0	-2.5	0.7	<0.001
4 semester	7.6	10.0	-2.5	0.7	<0.001
<b>Post-Secondary Math Passing: Developmental</b>					
1 semester	0.7	0.7	-0.1	0.2	0.805
2 semester	2.0	1.9	0.1	0.4	0.784
3 semester	2.0	1.9	0.1	0.4	0.792
4 semester	2.0	1.9	0.1	0.4	0.792

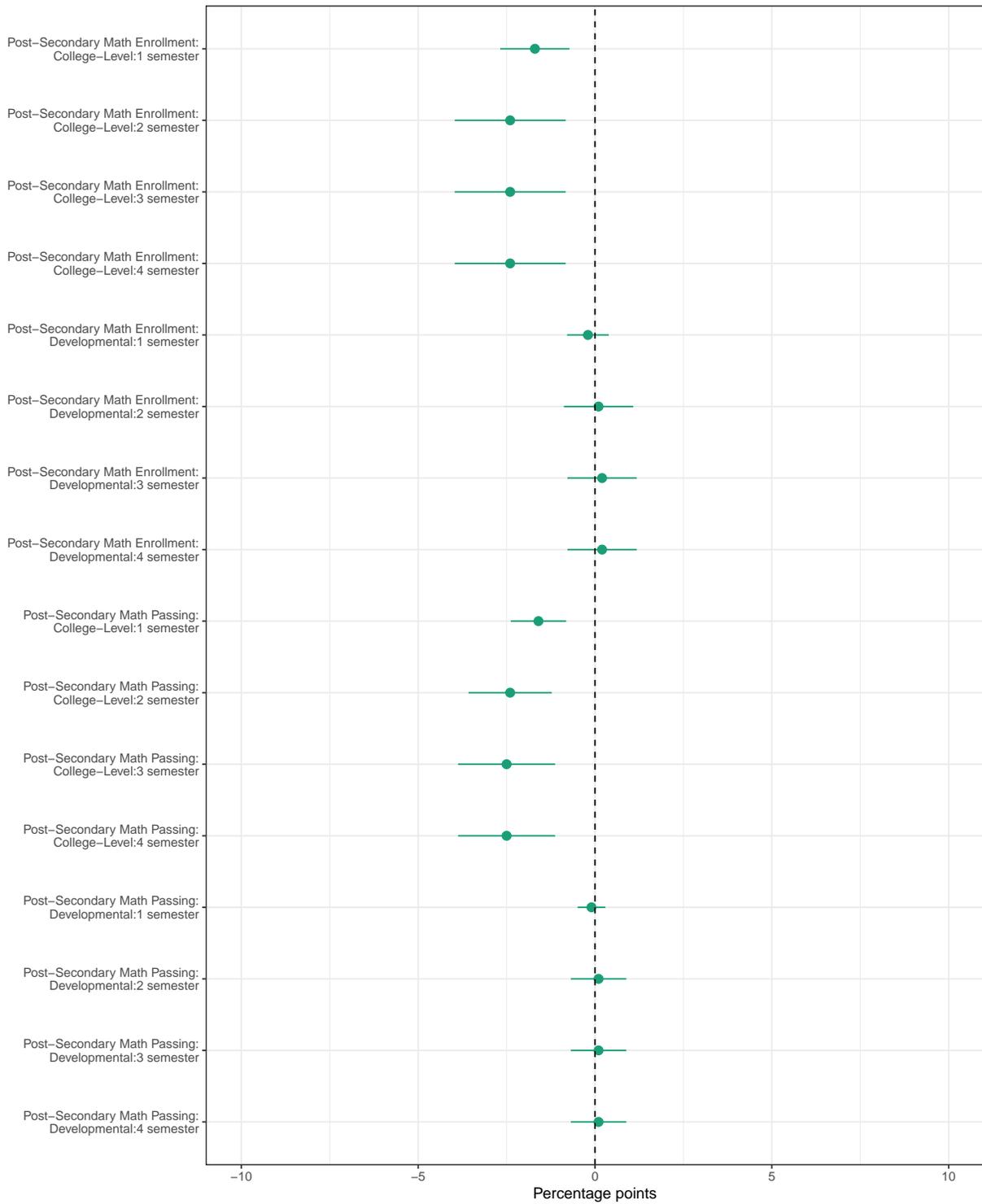


Figure 6. Overall average effects of taking TCMC on post-secondary math course-taking and course-passing rates. Dots represent point estimates. Lines correspond to 95% confidence intervals.

Figure 6 and Table 8 report the estimated effects of taking TCMC for post-secondary course-taking and course-passing outcomes. They are constructed in the same way as the previous Figure and Table. For these outcomes, there were consistent differences between TCMC students and comparison students in the rates of enrolling in and passing college-level math courses. By the fourth semester one year after high school graduation, 12.3 of students who took TCMC had enrolled in at least one college-level math course, compared to 14.8 of comparison students, a difference of -2.4 percentage points, 95% CI [-4.0, -0.8]. The differences in rates of passing at least one college-level math course within the same time frame show a very similar pattern, with 7.6 of students who took TCMC had passed at least one college-level math course, compared to 10.0 of comparison students, a difference of -2.5 percentage points, 95% CI [-3.9, -1.1]. In contrast, both students who took and TCMC and those in the comparison group enrolled in and passed developmental math courses at very similar rates.

### Discussion

To evaluate the effects of taking TCMC, we have compared the outcomes of students enrolled in the course during the 2017-18 school year to those of observationally similar peer students who did not participate in the course. Relative to the comparison group, we found that students who took TCMC graduated from high school at higher rates and enrolled at partner community colleges at a higher rates. Gains in community college enrollment rates were accompanied by declines in enrollment rates at four-year colleges and universities, so that the net change in enrollment rates was small and not statistically distinguishable from zero. Further, we found that students who took TCMC were less likely than comparison students to take or pass college-level math courses, with no differences in rates of taking or passing developmental math courses.

Overall, the pattern of findings suggests that TCMC could have shifted some students who might otherwise have enrolled in four-year institutions towards enrolling in

community college instead. This impact would be consistent with the incentive structure of the college preparatory course requirements created by HB 5, in that the successful completion of the course provided exemption from proficiency exams only at the partner institution. However, it is also possible that the pattern of results could be due to pre-existing differences between TCMC students and comparison students—particularly differences in aspirations or goals for attending four-year colleges. The incentive structure created by college preparatory math courses may have made it more likely for students to enroll in TCMC if their goal was to attend a partner community college than if their goal was to attend a four-year institution. If students enrolled in TCMC differed from comparison students in their aspirations to attend four-year institutions, even after accounting for differences in demographics, course-taking patterns, and prior academic performance, then the pattern of differences in post-secondary enrollment rates might be the result of bias rather than the impact of participating in TCMC.

Our previous analysis of the first year of TCMC implementation (2016-17) found mixed effects of TCMC on overall college enrollment rates, with the pattern of findings varying depending on whether effects were estimated using the contemporaneous comparison group or the previous-year comparison group. Estimates based on the contemporaneous comparison group indicated net reductions in overall college enrollment, driven by reduced enrollment in four-year institutions and only partially counter-acted by small gains in community college enrollment. In comparison, findings from the present analysis on the second year of TCMC implementation are more positive and consistent, with larger gains in community college enrollment and smaller (though still negative) reductions in four-year enrollment. One notable difference is that, among the 2016-17 cohort, only half of student who enrolled in community college selected the partnering institution, whereas among the 2017-18 cohort, over three quarters of students who enrolled in community college did so at the partnering institution.

Regarding college-level math course-taking and course-passing, findings from the

present analysis are consistent with the pattern of results from the analysis of the first year of implementation. In both cohorts, students who took TCMC showed lower rates of taking college-level math courses, as well as lower rates of passing such courses, relative to students in the comparison group. However, the magnitude of differences in course enrollment was smaller for the second cohort (-2.4 percentage points) than for the first cohort (-3.2 percentage points). Similarly, the magnitude of differences in course passage rates was smaller in the second cohort (-2.5, versus -6.0 percentage points for the first cohort), and more consistent with the differences in course enrollment rates. These discrepancies between the two cohorts might reflect improvements in the design or delivery of TCMC; however, it seems more likely that they indicate changes between cohorts in the population of students served by TCMC, as the course was offered in a larger and more diverse set of districts during the second year of implementation.

Just as in our analysis of the previous cohort, we must urge caution in interpreting these differences in enrollment as causal impacts of the program. Differences may have instead resulted from our inability to fully adjust for initial differences in college aspirations, as well as other potential confounders, at the start of students' senior year. Our analysis of the first year of implementation revealed that some results were sensitive to using the contemporaneous comparison group or the previous-year comparison, which may have indicated a degree of bias in our analytic approach. Given the sensitivity of findings from the first year, an important direction for further investigation is to add a second comparison group for estimating impacts on the second year of implementation.

Of particular concern is the possibility that the comparison group could have included some students who had already achieved college readiness by the start of senior year. Relative to students in TCMC, college-ready students in the comparison group could be expected to be more likely to apply to and gain admission into four-year colleges, which might explain the differences in four-year enrollment rates that we observed. The ideal approach to reducing remaining biases in the program impacts would be to account for

initial differences in college readiness. Unfortunately, we have been unable to data on initial college readiness status and it appears unlikely that we will be able to do so in the future.

The findings we have presented are limited to analysis of the overall population of students who took TCMC, aggregated across districts and schools. In ongoing work, we are examining the extent of heterogeneity in the effects across all schools in the second year of implementation and examining differences between schools that were implementing TCMC for the first time versus those who had offered the course the previous year.

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